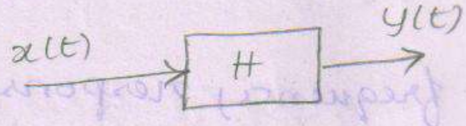
Frequency Response Analysis

Frequency Response Analysis :

Frequency Response :

It is the steady state response of the system when the input is sinusoidal signal.

Consider a linear time invariant system, H



Let,

$x(t) = X \sin \omega t$, be an sinusoidal signal.

ωt - phase angle.

The response $y(t)$ is also a sinusoidal signal of same frequency but with different magnitude and phase angle.

$$y(t) = Y \sin(\omega t + \phi)$$

The magnitude & phase relationship between the sinusoidal input and the steady state output of the system is termed as frequency response.

The frequency response of the system is obtained by varying the frequency of the input signal by keeping the magnitude of the input signal is constant.

In the system transfer function $T(S)$ if S is replaced by $j\omega$ $T(j\omega)$. Then the resultant $T(j\omega)$ is called sinusoidal transfer function.

The frequency response of the system can be directly obtained from the sinusoidal transfer function.

$T(j\omega)$ is a complex function of frequency. The frequency can be evaluated for both ~~bo~~ open loop and closed loop system.

Open loop T.F : $G(j\omega) = |G(j\omega)| \angle G(j\omega)$

Loop T.F : $G(j\omega)H(j\omega) = |G(j\omega)H(j\omega)| \angle G(j\omega)H(j\omega)$

Closed loop T.F: $\dfrac{C(j\omega)}{R(j\omega)} = M(j\omega) = |M(j\omega)| \angle M(j\omega)$

Frequency domain Specifications :-

The performance & characteristics of any system in frequency domain frequency domain Can be mentioned in terms of specifications.

any 1 definition 2 mark

(i) Resonant Peak, M_r

(ii) Resonant frequency, ω_r

(iii) Bandwidth, ω_b

(iv) cut off rate.

(v) Gain Margin, K_g

(vi) Phase Margin, γ

Resonant Peak M_r:

The maximum value of the magnitude of closed loop T.F is called resonant peak.

Resonant frequency, ω_r:

The frequency at which the resonant peak occurs is called resonant frequency.

Bandwidth, ω_b:

It is the range of frequencies, for which the system gain is more than -3 decibel.

The frequency at which the gain is -3 decibel is called cut-off frequency.

Bandwidth is usually defined for closed loop system and transmits the signal whose frequency are less than the cut-off frequency.

Cut-off rate:

The slope of the log magnitude curve near the cut-off frequency is called cut off rate.

Gain Margin kg:

It is defined as the reciprocal of magnitude of open loop Transfer function at phase cross over frequency (ω_{pc})

~~kg~~ The frequency at which the phase of open loop T.F is -180° is called ω_{pc}

$$k_g = \frac{1}{|G(j\omega)|_{at\ \omega=\omega_{pc}}} \qquad \angle G(j\omega) = -180^\circ$$

$$k_g \text{ in db} = 20 \log k_g = 20 \log \frac{1}{|G(j\omega_{pc})|}$$

$$k_g = -20 \log |G(j\omega_{pc})|$$
(in db)

Phase Margin, γ:

Phase margin is defined as that amount of additional phase lag at the gain cross over frequency (ω_{gc}) required to bring the system to the verge of instability.

The frequency at which the magnitude of open loop T.F is unity (or) 0 decibel. is known as ω_{gc}.

It is obtained by adding $180°$ to the phase angle open loop T.F at gain cross over frequency.

$$\gamma = 180° + \phi_{gc}$$

when $\phi_{gc} = \angle G(j\omega)_{gc}$

Estimation of frequency domain Specification of Second order system:

Consider the CLTF of second order system,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Replace s by $j\omega$:

$$T(j\omega) = \frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

$$T(j\omega) = \frac{\omega_n^2}{-\omega^2 + 2\xi\omega_n j\omega + \omega_n^2}$$

$$= \frac{\cancel{\omega_n^2}}{\cancel{\omega_n^2}\left(\frac{-\omega^2}{\omega_n^2} + 2\xi j\frac{\omega}{\omega_n} + 1\right)}$$

$$= \frac{1}{1 + 2\xi\frac{\omega}{\omega_n}j - \frac{\omega^2}{\omega_n^2}}$$

$$T(j\omega) = \frac{1}{1 - \frac{\omega^2}{\omega_n^2} + j2\xi\frac{\omega}{\omega_n}}$$

Let, normalised freq, $u = \frac{\omega}{\omega_n}$

$$T(j\omega) = \frac{1}{1 - u^2 + j2\xi u}$$

$$= \frac{1}{\sqrt{(1-u^2)^2 + 4\xi^2u^2}} \angle -\tan^{-1}\left(\frac{2\xi u}{1-u^2}\right)$$

$$M = |T(j\omega)| = \left[\frac{1}{(1-u^2)^2 + 4\xi^2u^2}\right]^{1/2} = \left[(1-u^2)^2 + \ldots\right.$$

$$\alpha = \angle T(j\omega) = -\tan^{-1}\left(\frac{2\xi u}{1-u^2}\right) = -\tan^{-1}\left(\frac{2\xi \ldots}{1-\ldots}\right)$$

$M_r \rightarrow$ max. value of M,

We can determine the condition for getting maximum 'M' value.

$$\frac{dM}{du} = 0 \Big|_{\text{when } M = M_r,\ u = u_r}$$

$$\frac{dM}{du} = \frac{d}{du}\left[(1-u^2)^2 + 4\xi_1^2 u^2\right]^{-1/2}$$

$$= -\frac{1}{2}\left[(1-u^2)^2 + 4\xi_1^2 u^2\right]^{-3/2}\left[2(1-u^2)(-2u) + 8\xi_1^2 u\right]$$

$$= -\frac{\left[-4u(1-u^2) + 8\xi_1^2 u\right]}{2\left[(1-u^2)^2 + 4\xi_1^2 u^2\right]}$$

$$= -\frac{\left[-4u(1-u^2) + 8\xi_1^2 u\right]}{2\left[(1-u^2)^2 + 4\xi_1^2 u^2\right]^{3/2}}$$

$$\frac{dM}{du} = 0 \Big|_{u = u_r} \Rightarrow \frac{4u(1-u_r^2) - 8\xi_1^2 u_r}{2\left[(1-u_r^2)^2 + 4\xi_1^2 u_r^2\right]^{3/2}} = 0$$

$$u_r = \frac{\omega_r}{\omega_n}$$

$$4u_r(1-u_r^2) - 8\xi_1^2 u_r = 0$$

$$4u_r(1-u_r^2) = 8\xi_1^2 u_r$$

$$1-u_r^2 = \frac{8\xi_1^2 u_r}{4u_r}$$

$$1 - u_r^2 = 2\xi_1^2$$

$$u_r^2 = 1-2\xi_1^2$$

$$u_r = \sqrt{1-2\xi_1^2}$$ for getting Mr.

Sub u_r in the M equal then the we get Mr

$$M_r = \frac{1}{\left[(1-u^2)^2+4\xi_1^2u^2\right]^{1/2}}\Bigg|_{u=u_r}$$

$$M_r = \frac{1}{\left[(1-u_r^2)^2+4\xi_1^2u_r^2\right]^{1/2}}$$

$$\cancel{= \frac{1}{1-2u_r+u_r^4+1}}$$

$$= \frac{1}{\left[(1-1+2\xi_1^2)^2+4\xi_1^2(1-2\xi_1^2\right]^{1/2}}$$

$$= \frac{1}{\left[4\xi_1^4+4\xi_1^2-8\xi_1^4\right]^{1/2}}$$

$$M_r = \frac{1}{\left[4\xi_1^2-4\xi_1^4\right]^{1/2}}$$

$$= \frac{1}{\left[4\xi_1^2\left[1-\xi_1^2\right]\right]^{1/2}}$$

$$\boxed{M_r = \frac{1}{2\xi_1\sqrt{1-\xi_1^2}}}$$

$$u_r^2 = 1 - 2\xi_1^2$$

$$u_r = \sqrt{1 - 2\xi_1^2} \text{ for getting } M_r.$$

Sub u_r in the M equal then the we get M_r

$$M_r = \frac{1}{\left[(1-u^2)^2 + 4\xi_1^2 u^2\right]^{1/2}}\Bigg|_{u=u_r}$$

$$M_r = \frac{1}{\left[(1-u_r^2)^2 + 4\xi_1^2 u_r^2\right]^{1/2}}$$

$$\cancel{= \frac{1}{1 - 2u_r + u_r^4 + 1}}$$

$$= \frac{1}{\left[(1 - 1 + 2\xi_1^2)^2 + 4\xi_1^2 (1 - 2\xi_1^2\right]^{1/2}}$$

$$= \frac{1}{\left[4\xi_1^4 + 4\xi_1^2 - 8\xi_1^4\right]^{1/2}}$$

$$M_r = \frac{1}{\left[4\xi_1^2 - 4\xi_1^4\right]^{1/2}}$$

$$= \frac{1}{\left[4\xi_1^2\left[1 - \xi_1^2\right]\right]^{1/2}}$$

$$\boxed{M_r = \frac{1}{2\xi_1\sqrt{1 - \xi_1^2}}}$$

(ii) Resonance frequency ω_r :-

$$\omega_r = ?$$

$$u_r = \frac{\omega_r}{\omega_n}$$

$$\omega_r = u_r \omega_n$$

$$\boxed{\omega_r = \omega_n \sqrt{1-2\xi^2}}$$

(iii) Bandwidth, ω_b :

$$\omega_b = ?$$

$$u_b = \frac{\omega_b}{\omega_n}$$

When $u = u_b$, M of CL $= \frac{1}{\sqrt{2}} = -3$ db

$$\therefore M = \frac{1}{\sqrt{2}} = \left[\frac{1}{(1-u^2)^2+4\xi^2u^2}\right]^{1/2}$$

$$M = \frac{1}{\left[(1-u_b^2)^2+4\xi^2u_b^2\right]^{1/2}} = 1/\sqrt{2}$$

$$\sqrt{2} = \left[(1-u_b^2)^2+4\xi^2u_b^2\right]^{1/2}$$

Squaring

$$2 = (1-u_b^2)^2+4\xi^2u_b^2$$

$$2 = 1-u_b^4-2u_b^2+4\xi^2u_b^2$$

$$= 1-u_b^4+2u_b^2(2\xi^2-1)$$

$$1 = -u_b^4+2u_b^2(2\xi^2-1)$$

$$1 + u_b^4 - 2u_b^2 + 4\xi_1^2 u_b^2 - 2 = 0$$

$$u_b^4 - 2u_b^2 (1 - 2\xi_1^2) - 1 = 0$$

Let $u_b^2 = x$.

$$x^2 - 2x(1 - 2\xi_1^2) - 1 = 0$$

$$x = \frac{2(1-2\xi_1^2) \pm \sqrt{4(1-2\xi_1^2)^2 - 4(-1)}}{2}$$

$$= \frac{\cancel{2}(1-2\xi_1^2) \pm \cancel{2}\sqrt{(1-2\xi_1^2)^2 + 1}}{\cancel{2}}$$

$$= (1 - 2\xi_1^2) \pm \sqrt{1 + 4\xi_1^4 - 4\xi_1^2 + 1}$$

$$x = (1 - 2\xi_1^2) \pm \sqrt{2 + 4\xi_1^4 - 4\xi_1^2}$$

consider +ve sign,

$$x = (1 - 2\xi_1^2) + \sqrt{2 - 4\xi_1^2 + 4\xi_1^4}$$

But $u_b = \sqrt{x} = \left[1 - 2\xi_1^2 + \sqrt{2 - 4\xi_1^2 + 4\xi_1^4}\right]^{1/2}$

$$u_b = \frac{\omega_b}{\omega_n}$$

$\therefore \omega_b = \omega_n \cdot u_b$

$$\boxed{\omega_b = \omega_n \left[1 - 2\xi_1^2 + \sqrt{2 - 4\xi_1^2 + 4\xi_1^4}\right]^{1/2}}$$

Phase Margin (γ):

Consider OLTF $G(s) = \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$

(2nd order)

$s \to j\omega$

$$G(j\omega) = \frac{\omega_n^2}{j\omega(j\omega + 2\xi\omega_n)}$$

$$= \frac{\omega_n^2}{-\omega^2 + 2\xi\omega_n j\omega}$$

$$= \frac{\omega_n^2}{\omega_n^2\left[-\frac{\omega^2}{\omega_n^2} + j2\xi\frac{\omega}{\omega_n}\right]}$$

$$u = \frac{\omega}{\omega_n}$$

$$\therefore G(j\omega) = \frac{1}{(-u^2 + j2\xi u)}$$

$$|G(j\omega)| = \frac{1}{\sqrt{u^4 + 4\xi^2 u^2}}$$

$$\angle G(j\omega) = \frac{1}{\tan^{-1}\left(\frac{2\xi u}{-u^2}\right)}$$

$$\angle G(j\omega) = -\tan^{-1}\left(-\frac{2\xi}{u}\right)$$

$$\angle G(j\omega) = \tan^{-1}\left(\frac{2\xi}{u}\right)$$

At $\omega = \omega_{gc}$, Magnitude is unity.

Let $u_{gc} = \frac{\omega_{gc}}{\omega_n}$

Sub u by u_{gc}, magnitude is unity

i.e) At $u = u_{gc}$; $|G(j\omega)|_{\substack{\omega = \omega_{gc} \\ u = u_{gc}}} = 1$

$$\therefore \frac{1}{\sqrt{u^4 + 4\xi^2 u^2}} = 1$$

$$\Rightarrow \frac{1}{\sqrt{u_{gc}^4 + 4\xi^2 u_{gc}^2}} = 1$$

Squaring & cross multiply.

$$\Rightarrow 1 = u_{gc}^4 + 4\xi^2 u_{gc}^2$$

$$u_{gc}^4 + 4\xi^2 u_{gc}^2 - 1 = 0$$

Consider $x = u_{gc}^2$

$$x^2 + 4\xi^2 x - 1 = 0$$

$$x = \frac{-4\xi^2 \pm \sqrt{16\xi^4 + 4}}{2}$$

$$= \frac{-4\xi^2 \pm 2\sqrt{4\xi^4 + 1}}{2}$$

$$= \frac{2(-2\xi^2 \pm \sqrt{4\xi^4 + 1})}{2}$$

$$x = -2\xi^2 + \sqrt{4\xi^4 + 1}$$

$$x = \omega_{gc}^2$$

$$\omega_{gc} = \sqrt{x}$$

$$\therefore \omega_{gc} = \left[-2\xi^2 + \sqrt{4\xi^4+1}\right]^{1/2}$$

Phase Margin $\gamma = 180° + \phi_{gc}$

$$\phi_{gc} = \angle G(j\omega)_{\omega = \omega_{gc}}$$

$$\gamma = 180° + \angle G(j\omega)_{gc}$$

$$\gamma = 180° + \tan^{-1}\left(\frac{2\xi}{\omega_{gc}}\right)$$

$$\boxed{\gamma = 180° + \tan^{-1}\left(\frac{2\xi}{\left[-2\xi^2+\sqrt{4\xi^4+1}\right]^{1/2}}\right)}$$

For second order System Gain Margin is infinity.

Frequency Response plots:

Two methods.

(i) Analytical method.

(ii) Graphical method.

(i) Bode plot
(ii) Polar plot (Nyquist plot)
(iii) Nichols plot

} OL system.

(iv) M & N circles

Frequency response plot are used to determine

1) frequency domain specifications.

2) Used to study the stability of the system.

3) Used to adjust the gain of the system to satisfy the desired specification.

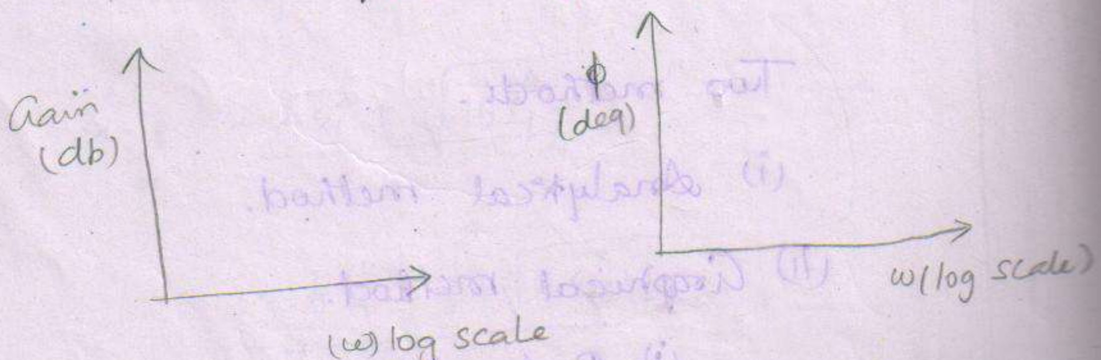
Bode plot:

↳ frequency response plot of sinusoidal transfer function of system.

↳ Mainly it is used for OL system only.

frequency response

Magnitude Plot | Phase angle plot.



Basic factor of $G(j\omega)$:

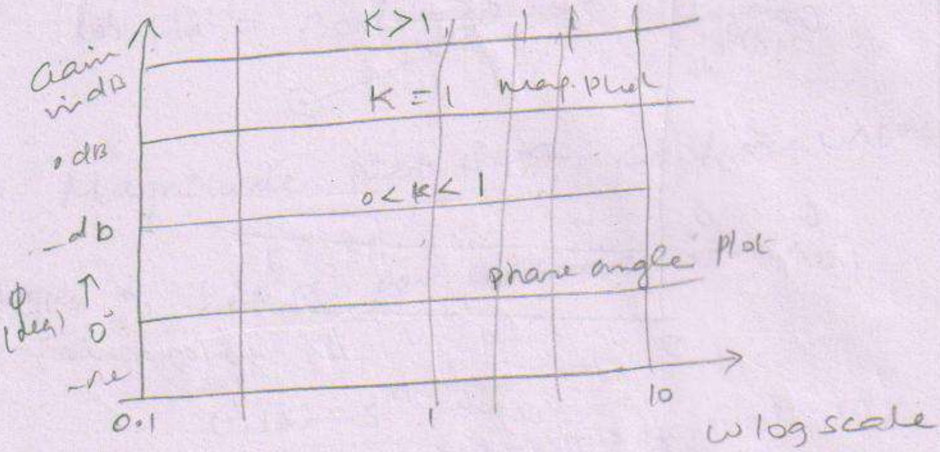
1) Constant Gain, K

$\therefore G(j\omega) = K = K + j0 = K\angle 0^\circ$

$|G(j\omega)| = K$

$\angle G(j\omega) = 0^\circ$

$|G(j\omega)|$ in db $= 20\log K$.



Integral factor:

$\frac{1}{j\omega}$ (or) $\frac{1}{(j\omega)^n}$

$20\log K$

$-20\log \omega$

Let $G(s) = \frac{1}{s}$

$\cancel{G(s)} = G(j\omega) = \frac{1}{j\omega}$

$|G(j\omega)| = \frac{1}{\sqrt{0^2+\omega^2}} = \frac{1}{\omega}$

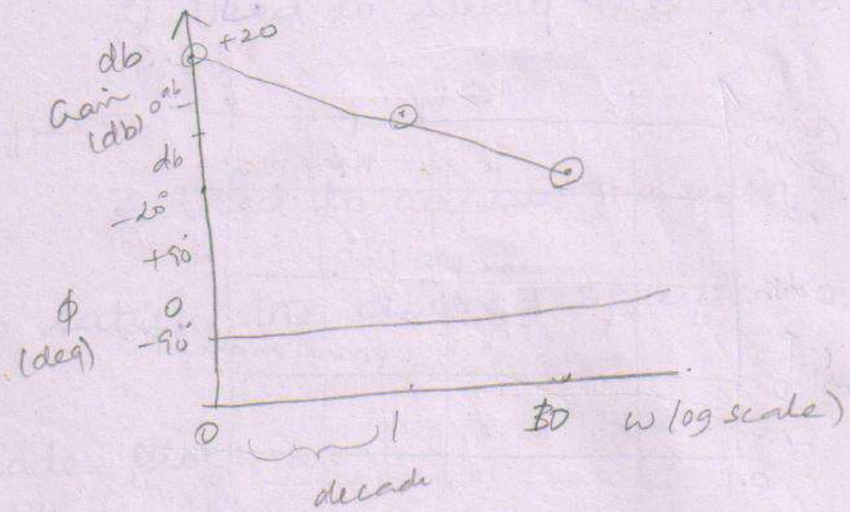
in db $= 20\log 1/\omega = -20\log \omega$

$\angle G(j\omega) = \frac{1}{\tan^{-1}(\omega/0)} = -\tan^{-1}\infty$

$= -90^\circ$

Magnitude plot is a ~~slope~~ st. line

with a slope of -20 db per decade.



$$G(s) = \frac{1}{s^2}$$

$$G(j\omega) = \frac{1}{j\omega j\omega}$$

$$|G(j\omega)| = \frac{1}{\sqrt{0^2 + (\omega^2)^2}} = \frac{1}{\omega^2}$$

$$\text{in db} = 20\log 1/\omega^2$$

$$= -20\log \omega^2$$

$$= -40\log\omega$$

In general,

$$|G(j\omega)| \text{ in db} = -20\, n \cdot \log\omega$$

$$= 1/s^n.$$

$$\angle G(j\omega) = -90 \times n$$

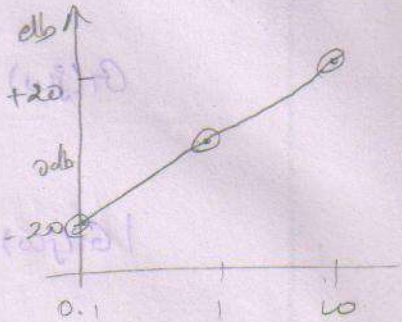
$$\angle G(j\omega) = -\tan^{-1}\ldots$$

$$= -90.9\ldots$$

(iii) Derivative factor:

$$G(s) = s \text{ (or) } s^n$$

$$G(j\omega) = j\omega \text{ (or) } (j\omega)^n$$



$$|G(j\omega)| = \sqrt{\omega^2} = \omega$$

$$\text{in db} = 20 \log \omega$$

Magnitude plot is a st. line with a slope of +20 db per decade.

$$G(s) = s^n$$

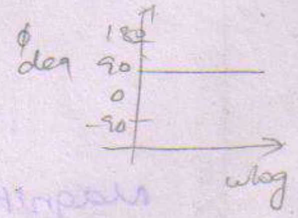
$$G(j\omega) = (j\omega)^n$$

$$|G(j\omega)| = \omega^n$$

$$\text{in db} = 20 \log \omega^n \Rightarrow 20\, n \log \omega.$$

$G(j\omega) = j\omega$

$$\underline{|G(j\omega)} = \tan^{-1}(\omega/0) = \tan^{-1}(\infty) = 90°$$



$$G(s) = s^2$$

$$G(j\omega) = j\omega \cdot j\omega$$

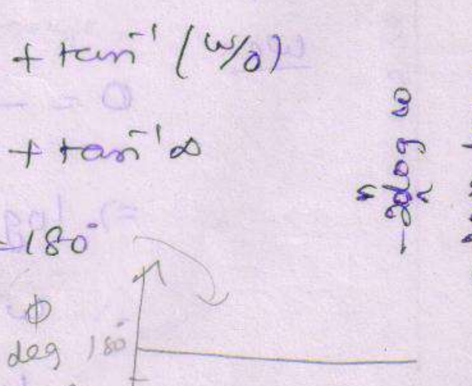
$$\underline{|G(j\omega)} = \tan^{-1}\left(\frac{\omega}{0}\right) + \tan^{-1}(\omega/0)$$

$$= \tan^{-1}\infty + \tan^{-1}\infty$$

$$= 90° + 90° = 180°$$

$-20 \log \omega^n$ $\quad -20\, n \log \omega$ $\quad -90° \times n$

$$G(j\omega) = (j\omega)^n$$



$$\underline{|G(j\omega)^n} = 90° \times n.$$

$= -90° \times n$

4) First order factor in denominator :-

$$G(s) = \frac{1}{1+sT}$$

$$G(j\omega) = \frac{1}{1+j\omega T}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2T^2}}$$

$$\text{in db} = 20\log\frac{1}{\sqrt{1+\omega^2T^2}}$$

$$= -20\log\sqrt{1+\omega^2T^2}$$

for low frequency range,

$$\omega T << 1$$

$$\text{Magnitude in db} \simeq -20\log 1$$

$$= 0\,\text{db.}$$

for high frequency range,

$$\omega T >> 1$$

$$\text{Magnitude in db} = \cancel{20} - 20\log\omega T$$

$\underline{\omega_c}$:

$$0 = -20\log\omega T$$

$$\Rightarrow \log\omega T = 0$$

$$\therefore \omega T = 1$$

$$\omega = \frac{1}{T}$$

$$\omega_c = \frac{1}{T}$$

(corner frequency)

Upto $\omega_c = \frac{1}{T}$ the magn. is 0 db.

above $\omega_c = \frac{1}{T}$ the magnitude is ~~-20~~ $-20\log\omega T$

Phase angle :

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega T}{1}\right)$$

$$= -\tan^{-1}(\omega T)$$

5) First order factor in Numerator :-

$$G(s) = 1 + sT$$

$$G(j\omega) = 1 + j\omega T$$

$$|G(j\omega)| = \sqrt{1+\omega^2T^2}$$

$$\text{in db} = 20\log\sqrt{1+\omega^2T^2}$$

for low frequency range.

$$\omega T << 1$$

$$\text{Mag. in db} \simeq 20\log 1$$

$$= 0 \text{ db.}$$

ωT $-2\log\omega$

for high frequency range,

$$\omega T >> 1$$

$$\text{Mag. in db} = 20\log\omega T$$

$\underline{\underline{\omega_c}}$:

$$0 = 20\log\omega T$$

$$\log\omega T = 0$$

$$\omega = \frac{1}{T}$$

Upto $\omega c = \frac{1}{T}$ the mag. is 0 db

Above $\omega c = \frac{1}{T}$ the mag is $20 \log \omega T$

Phase angle:

$$\angle G(j\omega) = \tan^{-1}\left(\frac{\omega T}{1}\right)$$

$$= \tan^{-1}(\omega T)$$

6) Quadratic factor in denominator:

$(2s^2+3s+4)$

$$G(s) = \frac{\omega n^2}{s^2 + 2\xi \omega n s + \omega n^2}$$

$$= \frac{\cancel{\omega n^2}}{\cancel{\omega n^2}\left(1 + \frac{2\xi}{\omega n} s + \frac{1}{\omega n^2} s^2\right)}$$

$$= \frac{1}{1 + \frac{2\xi}{\omega n} j\omega + \frac{(j\omega)^2}{\omega n^2}}$$

$$G(j\omega) = \frac{1}{1 + j\frac{2\xi\omega}{\omega n} - \frac{\omega^2}{\omega n^2}}$$

$$= \frac{1}{\left(1 - \frac{\omega^2}{\omega n^2}\right) + j\frac{2\xi\omega}{\omega n}}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega n^2}\right)^2 + \frac{4\xi^2\omega^2}{\omega n^2}}}$$

in db $= 20 \log \frac{1}{\ldots}$

$$= -20\log \sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \frac{4\xi^2\omega^2}{\omega_n^2}}$$

for low frequency range:

$$\omega << \omega_n \qquad \frac{\omega}{\omega_n} << 1.$$

$$\text{Mag in db} = -20\log 1 = 0\,db$$

for high frequency range.

$$\omega >> \omega_n \qquad \frac{\omega}{\omega_n} >> 1.$$

$$\text{Mag in db} = -20\log \sqrt{\left(-\frac{\omega^2}{\omega_n^2}\right)^2}$$

$$= -20\log \frac{\omega^2}{\omega_n^2}$$

$$= -20\log \left(\frac{\omega}{\omega_n}\right)^2$$

$$= -40\log \left(\frac{\omega}{\omega_n}\right)$$

Corner frequency:

$$\omega_n = \omega_c = \omega_n$$

$$0 = -40\log\left(\frac{\omega}{\omega_n}\right)$$

$$\log(\omega/\omega_n) = 0$$

$$\frac{\omega}{\omega_n} = 1$$

$$\Rightarrow \omega = \omega_n$$

$$\therefore \boxed{\omega_c = \omega_n}$$

$(2s^2+3s+4)$

Phase angle:

$$\angle G(j\omega) = -\tan^{-1}\left[\frac{2\xi\,\omega/\omega_n}{1-\omega^2/\omega_n^2}\right]$$

Upto $\omega_c = \omega_n$ the mag is 0 db.

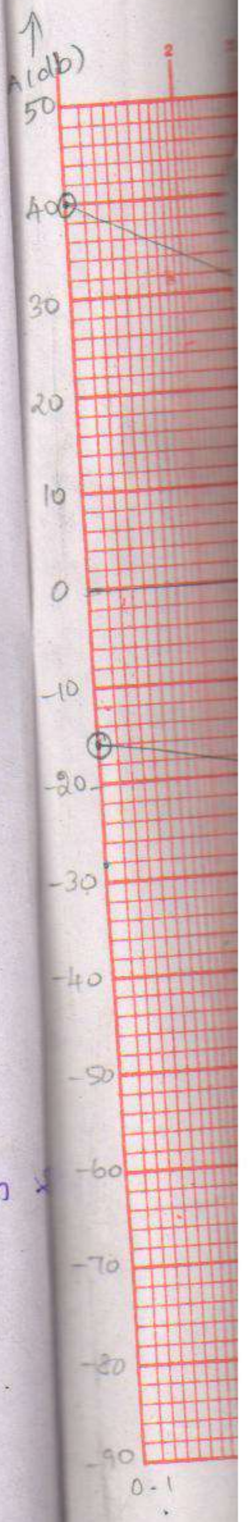
Above $\omega_c = \omega_n$ the mag is $-40\log\frac{\omega}{\omega_n}$

Second order factor in Numerator:

$$m\,db = 20\log\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \frac{4\xi^2\omega^2}{\omega_n^2}}$$

Upto $\omega_c = \omega_n$ the mag is 0db.

~~Upto~~ ~~to~~ Above, $\omega_c = \omega_n$, the mag is 40 ~~dc~~ $\log\frac{\omega}{\omega_n}$.



1). plot the Bode diagram & obtain the gain & phase cross over frequency

$$G(s) = \frac{10}{s(1+0.4s)(1+0.1s)}$$

Sol

$\frac{C(s)}{R(s)} = 1$

$\frac{G(s)}{1+G(s)H(s)}$

To get Sinusoidal T.F

$s = j\omega$

$$G(j\omega) = \frac{10}{j\omega(1+0.4j\omega)(1+0.1j\omega)}$$

for. $\omega c_1 = \frac{1}{T} = \frac{1}{0.4} = 2.5$ rad/s

$\omega c_2 = \frac{1}{T} = \frac{1}{0.1} = 10$ rad/s

ω	K=1 const. fac Mg = 20log 10	1/s mag = −20log ω	ωc = 2.5 $\frac{1}{1+0.4s}$ 0db / −20log 0.4	ωc = 10 $\frac{1}{1+0.1s}$ 0 / −20log (0.1)	Resultant mag in db
0.1	20	20	0	0	40
1.0	20	0	0	0	20
2	20	−6.021	0	0	14
5	20	−14 ~~13.97~~	−6.02	0	0
10	20	−20	−12	0	−12
50	20	−34	−26	−14 ~~13.9~~	−54
100	20	−40	−32	−20	−72
500	20.	−54	−46	−34	−114

$$\phi = -90^\circ - \tan^{-1}(0.4\omega) - \tan^{-1}(0.1\omega)$$

ω	0.1	1	2.5	4	10	20
φ (deg)	−93	−118	−150	−170	−210	−236

2) $G(s) = \frac{20}{s(1+3s)(1+4s)}$

Sol

$s = j\omega$

$G(j\omega) = \frac{20}{j\omega(1+3j\omega)(1+4j\omega)}$

$\omega_1 = \frac{1}{3} = 0.333$ rad/s $\quad \omega c_2 = \frac{1}{4} = 0.25$ rad/s

ω	K = 20 const Mag = 20log20	1/s Mag = −20 log ω	$\omega_{c1} = 0.2$ $\frac{1}{1+4s}$ 0 dB / −20log 4ω	$\omega_{c2} = 0.3$ $\frac{1}{1+3s}$ 0 dB / −20log 3ω	Resultant magnitude
0.01	26	−40	28	30	44
0.1	26	−20	0	0	6
1	26	0	−12	−9	5
1.1	26	0.827	−13	−10	3.827
1.2	26	−2	−14	−11	−1
2	26	−6	−18	−16	−14
5	26	−14	−26	−24	−38
10	26	−20	−32	−30	−56
50	26	−34	−51	−44	−103

$$\phi = -90° - \tan^{-1}(4\times\omega) - \tan^{-1}(3\times\omega)$$

ω	0.1	1	0.2	0.3	0.31	0.29
φ	−129	−238	−160	−182	−184	−180

3) Sketch the Bode plot for the following T.F and determine phase margin and gain Margin.

$$G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)}$$

Sol

$$= \frac{75(1+0.2s)}{s\,100\left(\frac{s^2}{100} + \frac{16}{100}s + 1\right)}$$

$$= \frac{0.75(1+0.2s)}{s(1+0.16s+0.01s^2)}$$

$s = j\omega$

$$G(j\omega) = \frac{0.75(1+0.2j\omega)}{(j\omega)\left(1+0.16j\omega+0.01(j\omega)^2\right)}$$

$$= \frac{0.75(1+0.2j\omega)}{j\omega\left[(1-0.01\omega^2)+0.16j\omega\right]}$$

$$k_g = 20\log\frac{1}{|G(j\omega_{pc})|}$$

(i) $\omega c_1 = \frac{1}{T} = \frac{1}{0.2} = 5$ rad/s

for Quadratic factor

$\omega_c = \omega_n$

$s^2+16s+100 \Rightarrow s^2+2\xi\omega_n s+\omega_n^2$

$\omega_n^2 = 100$

$\boxed{\omega_n = 10}$

(ii) $\omega c_2 = 10$ rad/s

ω	const $20\log 0.75$	$1/j\omega$ $-20\log\omega$	$\omega c_1 = 5$ rad $1+0.2\omega$ 0 db / $20\log 0.2\omega$	Quadratic in deno $\omega_c=10$ 0 db / $-40\log\frac{\omega}{10}$ $-40\log 0.1\omega$	Resultant Mag. (in db)
0.1	-2.5	20	0	0	17.5
1	-2.5	0	0	0	-2.5
5	-2.5	-14	0	0	-16.5
10	-2.5	-20	6.02	0	-16.5
50	-2.5	-34	20	-28	-44.5
100	-2.5	-40	26	-40	-56.5
0.2	-2.5	14	-28	68	51.5
0.9	-2.5	1	-15	42	25

Phase angle:

$$\phi = \tan^{-1}(0.2\omega) - 90^\circ - \tan^{-1}\left[\frac{0.16\omega}{1-0.01\omega^2}\right]$$

$\Rightarrow \omega \le \omega_n$

$$\phi = \tan^{-1}(0.2\omega) - 90^\circ - \left[\tan^{-1}\left(\frac{0.16\omega}{1-0.01\omega^2}\right) + 180\right]$$

$\Rightarrow \omega > \omega_n$

$= \tan^{-1}(\infty) - 90^\circ - (\tan^{-1}(0) + 180^\circ]$ 179.12

ω	0.1	1	5	10	50	100	
ϕ	-89	-88	-92	-87	-167.3	~~-182~~	
	-88	-88	-92	-117	-168	-174	-269
				20	1000	∞	$180 - 269$
				-147	-179	-180°	

Phase Margin = -88° $PM = 180^\circ + \phi_{gc}$

$= 180^\circ - 88$

Gain Margin = ∞ $\boxed{4.66\ HW}$ $PM = 92^\circ$

1) Sketch the bode plot for the following T.F & determine the system gain K for the gain cross over frequency for 5 rad/sec.

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)}$$

Sol

Unit - IV completed 29/10/16