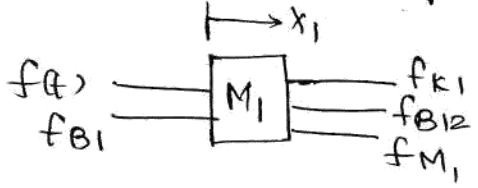
Step 1:-

Draw free body diagram,



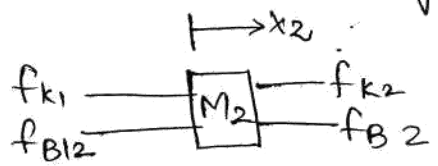
By using Newtons II law,

$$f(t) = f_{M1} + f_{K1} + f_{B12} + f_{B1}$$

$$\boxed{f(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + k_1 (x_1 - x_2) + B_{12} \frac{d}{dt}(x_1 - x_2)} \rightarrow (1)$$

Step 2:

Draw free body diagram,



By Newtons II law,

$$f_{M2} + f_{k2} + f_{B2} + f_{k1} + f_{B12} = 0.$$

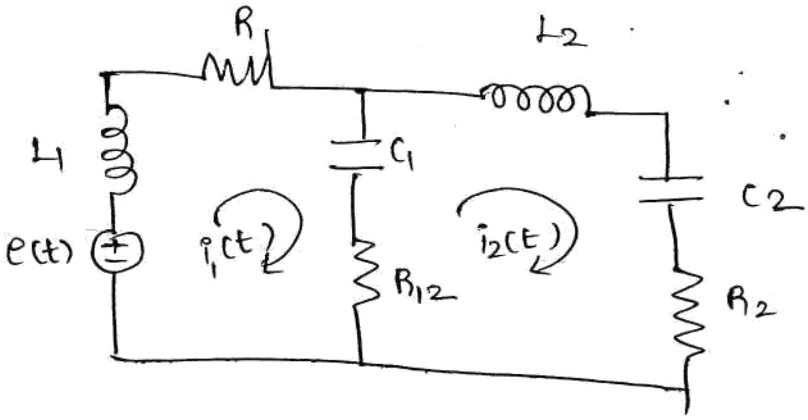
$$\boxed{M_2 \frac{d^2 x_2}{dt^2} + K_2 x_2 + B_2 \frac{dx_2}{dt} + K_1 (x_2 - x_1) + B_{12} \frac{d}{dt}(x_2 - x_1) = 0} \rightarrow (2)$$

Step 3: Force - Voltage Analogy.

$f(t) = e(t)$ $\quad M_1 = L_1$ $\quad B_1 = R_1$ $\quad k_1 = 1/C_1$

$V_1 = i_1(t)$ $\quad M_2 = L_2$ $\quad B_2 = R_2$ $\quad k_2 = 1/C_2$

$V_2 = i_2(t)$ $\quad B_{12} = R_{12}$



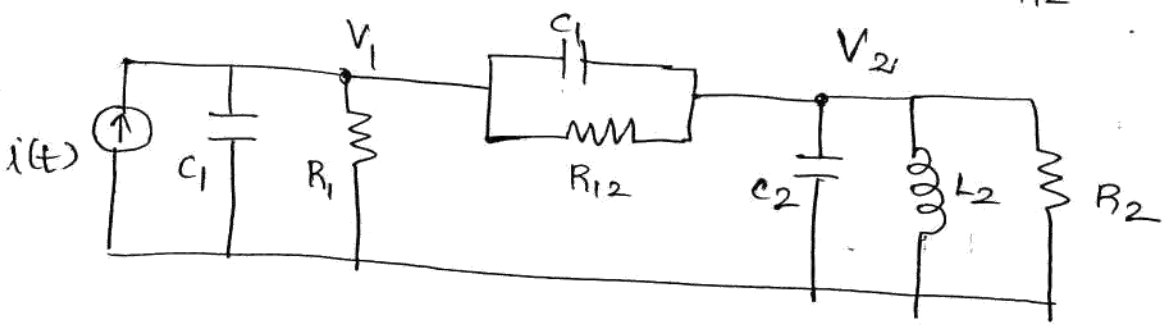
Step 4:- Force current Analogy system.

$f(t) = \hat{i}(t)$ $\quad M_1 = C_1 \quad K_1 = 1/L_1 \quad B_1 = 1/R_1$

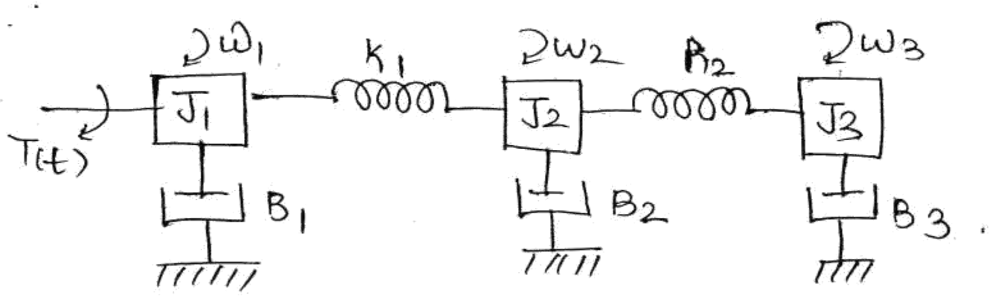
$V_1 = V_1(t)$ $\quad M_2 = C_2 \quad K_2 = 1/L_2 \quad B_2 = 1/R_2$

$V_2 = V_2(t)$

$B_{12} = 1/R_{12}$.



3. Draw the Torque-Voltage & Torque current Analogous System for the mechanical system.



Step 1:-

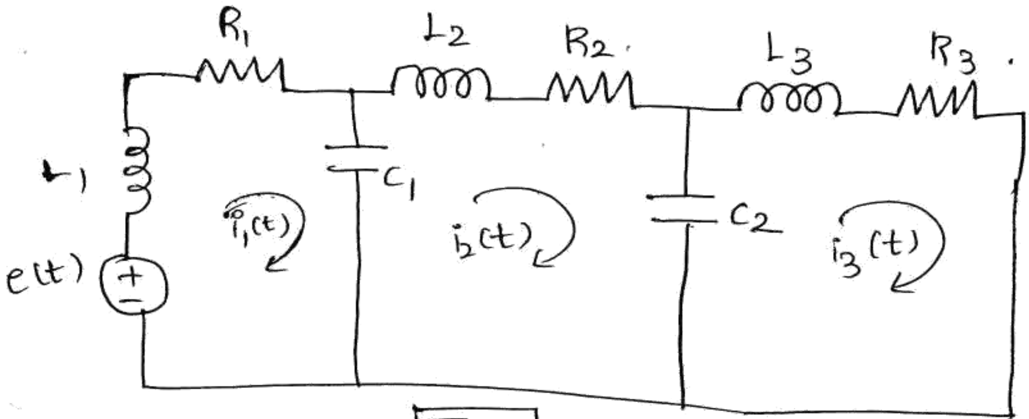
Identify equivalent electrical systems ~~in terms of Voltage~~.

$T(t) = e(t)$ $\quad J_1 = L_1$ $\quad K_1 = 1/C_1$ $\quad B_1 = R_1$

$\omega_1 = i_1(t)$ $\quad J_2 = L_2$ $\quad K_2 = 1/C_2$ $\quad B_2 = R_2$

$\omega_2 = i_2(t)$ $\quad J_3 = L_3$ $\quad B_3 = R_3$

$\omega_3 = i_3(t)$



$\boxed{T-V}$ Analogous s/m

Step 2:-

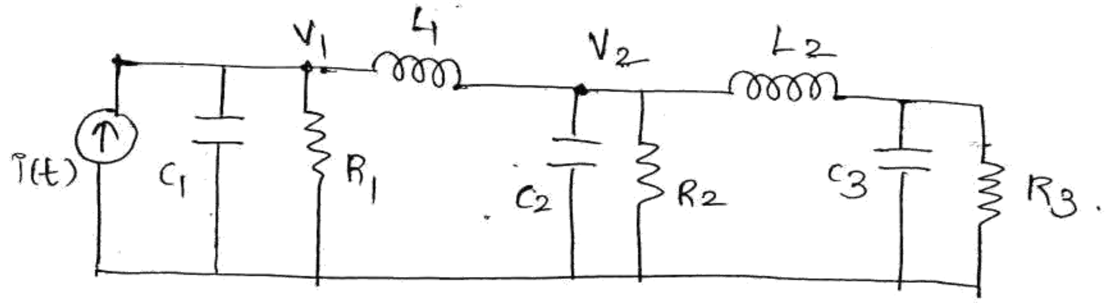
Identify equivalent electrical system in terms of current.

$T(t) = i(t)$ $\quad J_1 = C_1$ $\quad K_1 = 1/L_1$ $\quad B_1 = 1/R_1$

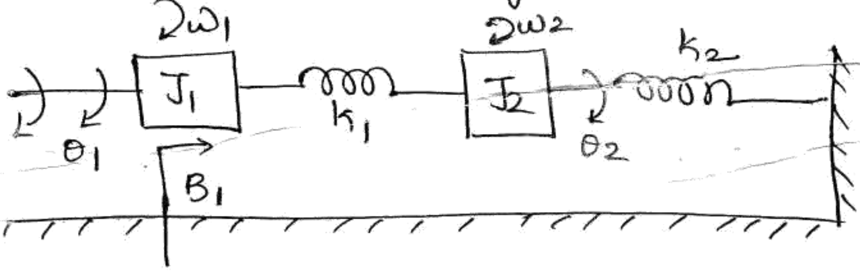
$\omega_1 = V_1$ $\quad J_2 = C_2$ $\quad K_2 = 1/L_2$ $\quad B_2 = 1/R_2$

$\omega_2 = V_2$ $\quad J_3 = C_3$ $\quad B_3 = 1/R_3$

$\omega_3 = V_3$

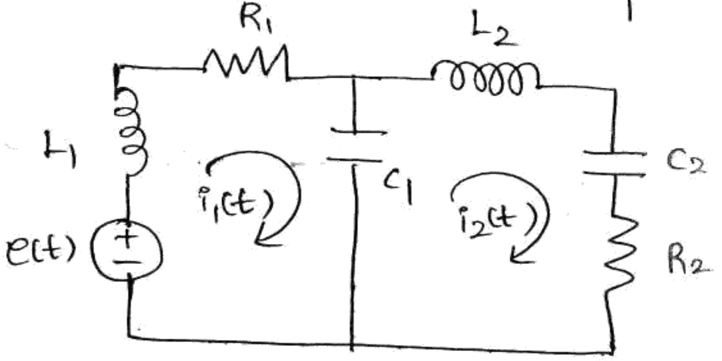


2. Draw the torque Voltage ... torque current analogy System for the following mechanical rotational s/m.



Step 1:- Draw Torque - Voltage circuit diagram.

$T = e(t)$	$J_1 = L_1$	$K_1 = 1/C_1$	$B_1 = R_1$
$\omega_1 = i_1(t)$	$J_2 = L_2$	$K_2 = 1/C_2$	$B_2 = R_2$
$\omega_2 = i_2(t)$			

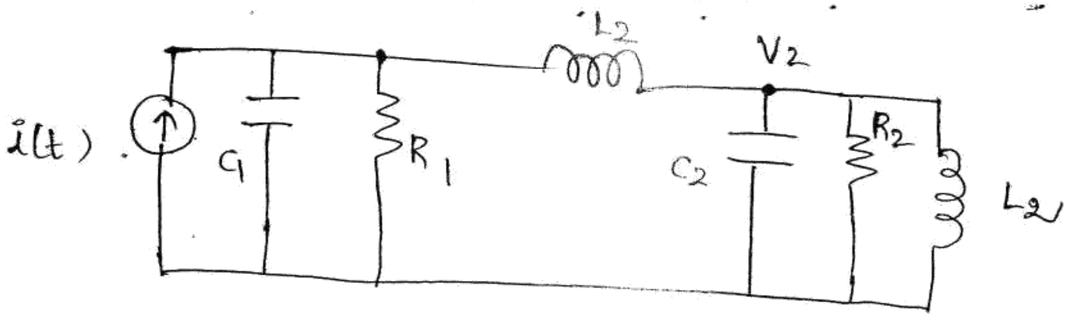


Step 2:- Draw Torque - current circuit diagram.

$T = i(t)$ $\qquad J_1 = C_1$ $\qquad K_2 = 1/L_2$

$\omega_1 = V_1(t)$ $\qquad J_2 = C_2$ $\qquad B_1 = 1/R_1$

$\omega_2 = V_2(t)$ $\qquad K_1 = 1/L_1$ $\qquad B_2 = 1/R_2$



BLOCK DIAGRAM REDUCTION TECHNIQUE

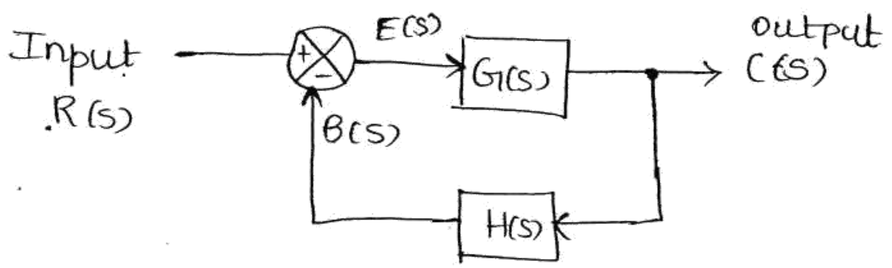
Any control system will have a number of control components. A control system can be represented in Block diagram form. The arrow head pointing towards a particular block indicates input to the system component and arrow head leading away from the block indicates output.

Block Diagram is possible to evaluate the contribution of each of components towards overall performance of control system.

It helps in understanding functional operation of the system more readily than examination of actual control system physically.

It may be noted that Block diagram drawn for a system is not unique.

Block diagram representation of closed loop:-



Block diagram is also called canonical form.

$$E(s) = R(s) - B(s)$$

$$B(s) = H(s).C(s)$$

$$\boxed{C(s) = E(s)G(s)}$$

$$R(s) = E(s) + B(s)$$
$$= E(s) + H(s)C(s)$$
$$= E(s) + H(s)E(s)G(s).$$

$$\therefore \boxed{R(s) = E(s) + H(s)E(s)G(s)}$$

$\therefore$ closed loop Transfer function $\Big\}$ $\dfrac{C(s)}{R(s)} = \dfrac{E(s)G(s)}{E(s) + H(s)E(s)G(s)}$

($\div$ by $E(s)G(s)$ on N_r & D_r) $= \dfrac{1}{\frac{1}{G(s)} + H(s)}.$

$$T.F = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \longrightarrow \text{Negative feedback}$$

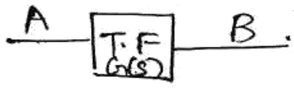
$$T.F = \frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)} \longrightarrow \text{positive feedback.}$$

$$TF = \frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)} \longrightarrow \text{unity feedback}$$
(ie., $H(s) = 1$)

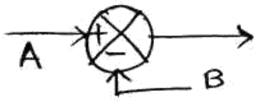
———— x ————

Rules for Block Diagram simplification

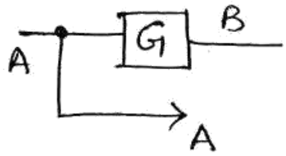
There are some rules which helps to simplify a block diagram of control system and there are three basic elements in Block diagram.

(i) Block 

$\boxed{B = AG(s)}$

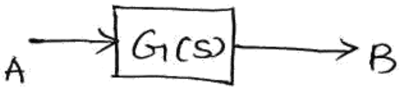
(ii) Summing point 

$\boxed{A - B}$

(iii) Branching point. 

$\boxed{B = AG}$

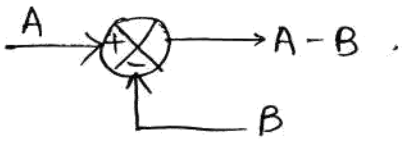
<u>Block</u>: It is a symbol for mathematical operation on the input signal to the block that produces the output.



$\boxed{B = AG(s)}$

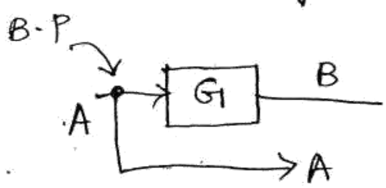
<u>Summing point</u>:

It is used to add two or more signals in the system.



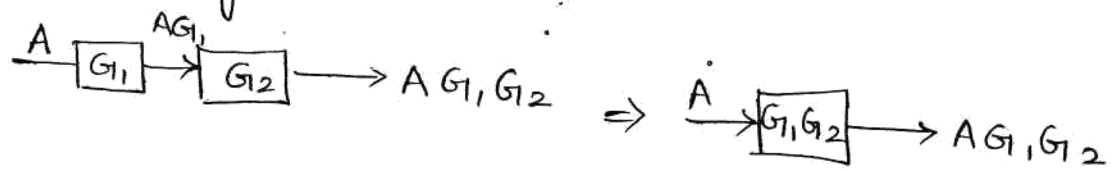
<u>Branch point</u>:

It is a point from which the signal from a block goes concurrently to other blocks or summing points.

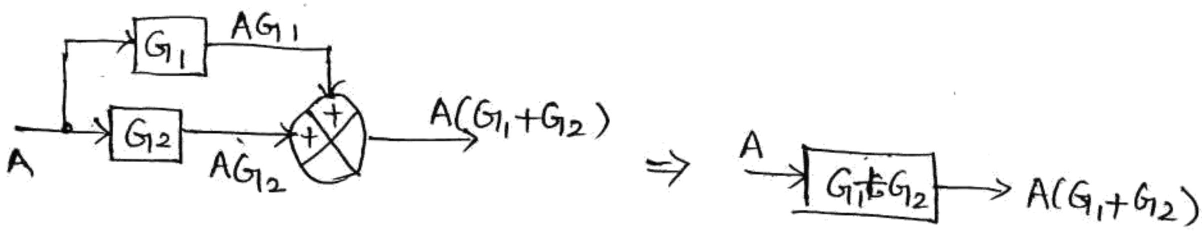


$\boxed{B = AG}$

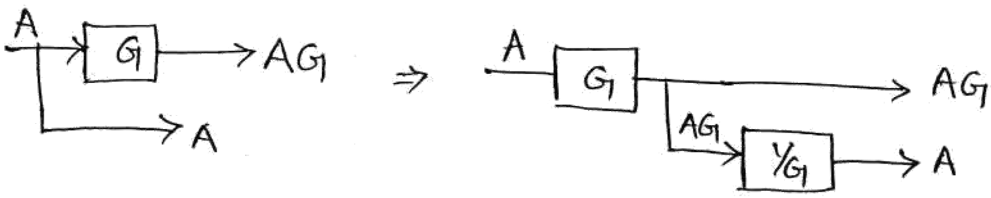
Rule 1: Combining the blocks in cascade



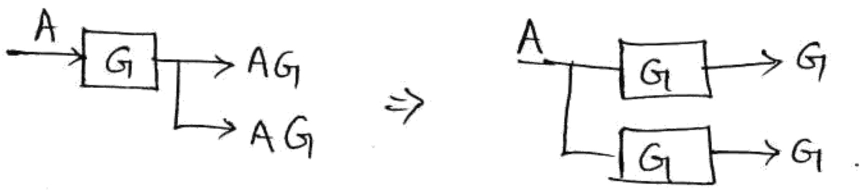
Rule 2: combining parallel blocks (or combining feed forward paths).



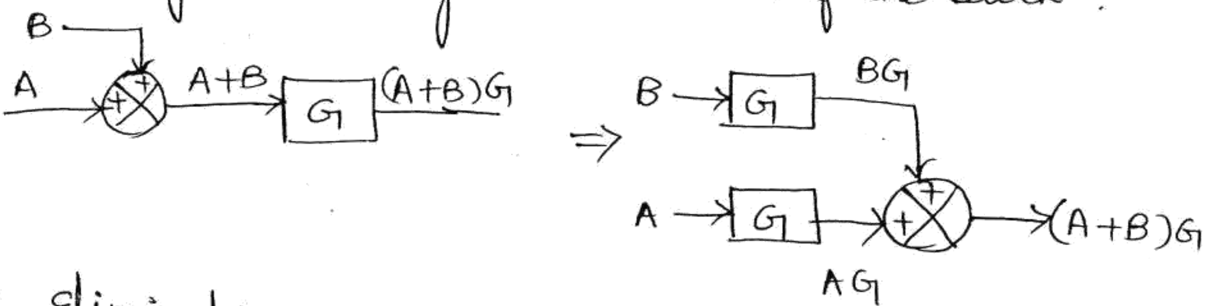
Rule 3: Moving the branch point ahead of block.



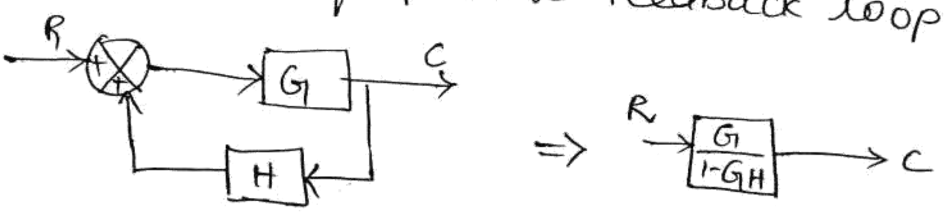
Rule 4: Moving branch point before the block.



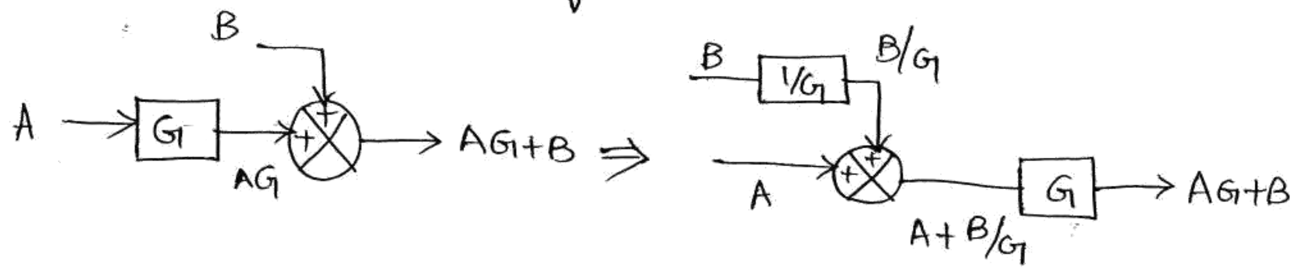
Rule 5: Moving summing point ahead of the block.



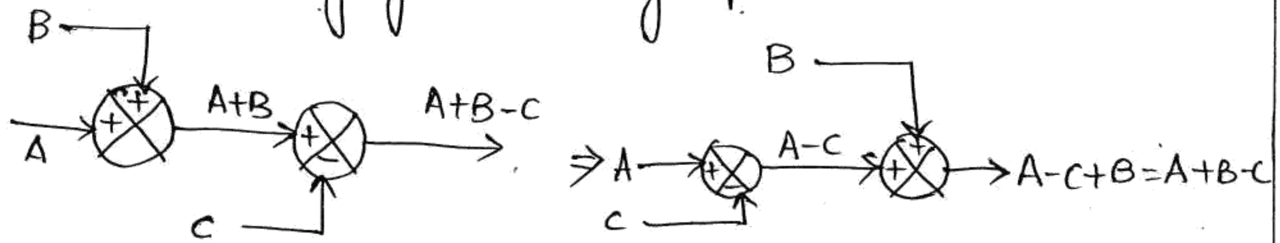
Rule 6: Elimination of positive feedback loop.



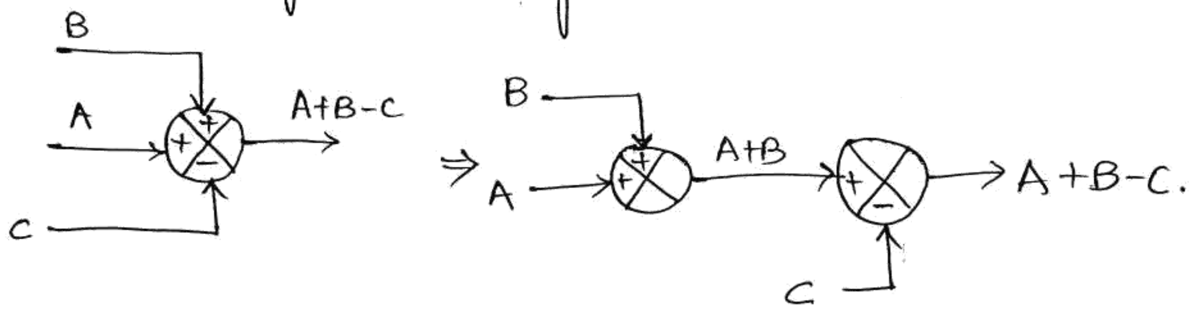
Rule 7:- Moving summing point before the block.



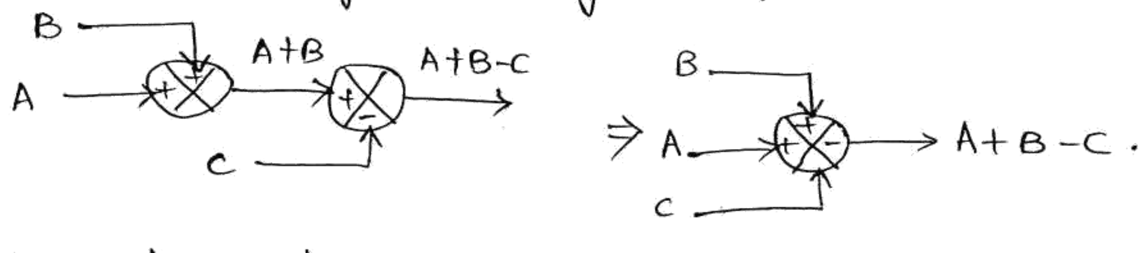
Rule 8: Interchanging summing point.



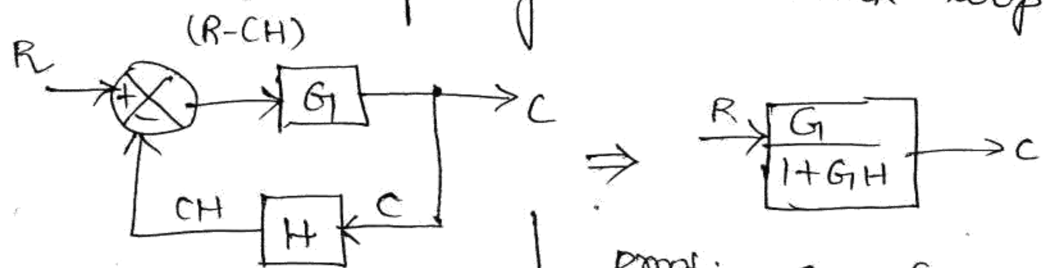
Rule 9: splitting summing points.



Rule 10: Combining summing Blockpoints.



Rule 11: Elimination of negative feedback loop.



$C = (R - CH)G$

$C + CHG = RG \Rightarrow \frac{C}{R} = \frac{G}{1+GH}$

Proof: $\frac{C}{R} = \frac{G}{1+GH}$

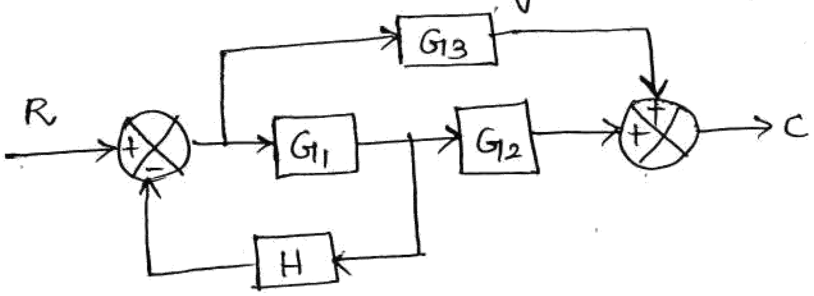
Note: * Branch points & summing points cannot be interchanged.

* No square terms in the Transfer function.

—— × —— × ——

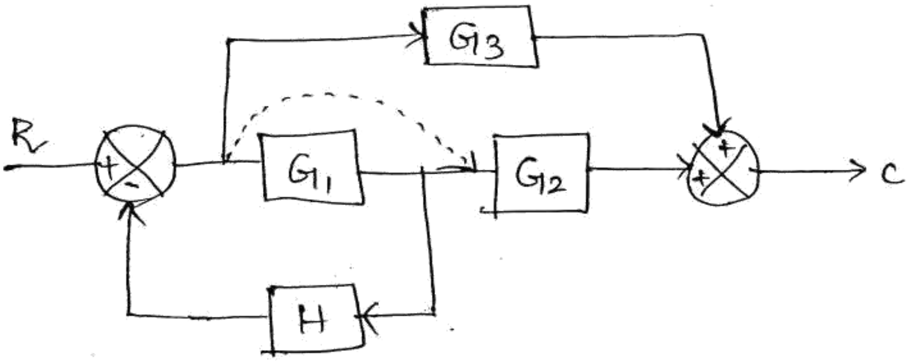
Problems

1. Reduce the block diagram and find C/R.



Solution:

step 1: Move the branch point after the block,



SIGNAL FLOW GRAPH MODEL:

For complicated systems, Block diagram reduction approach for arriving at transfer function relating the input and output Variables is tedious and time consuming.

signal Flow graph (SFG) is an alternative approach developed by "S.J. Mason". It does not require any reduction process because of availability of flow graph gain formula which relates input and output system Variables.

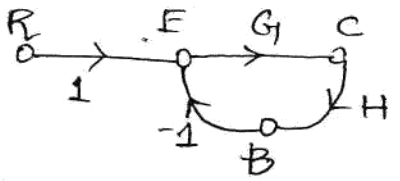
Definition:

A SFG is a graphical representation of relationship between the Variables of set of linear algebraic equations. It consists of a network in which nodes representing each of system Variables are connected by directed branches.

Some important terms in SFG are as follows

i) Node:

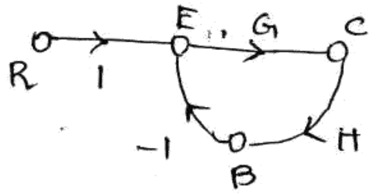
It represents a system Variable which is equal to sum of all incoming signals at the node. outgoing signals from node do not affect the Value of node Variable.



Here, R, E, C are nodes.

ii) Branch :-

A signal travels along a branch from one node to another in direction indicated by the branch arrow and in process, gets multiplied by gain or transmittance.

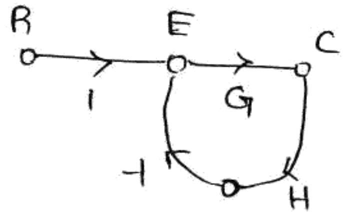


⇒ here, G is branch &

$\boxed{C = GE}$

iii) Input node or Source :-

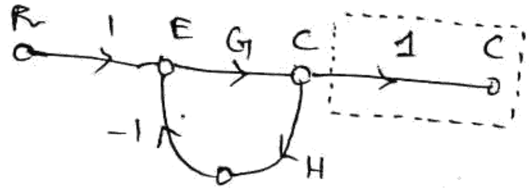
It is a node with only outgoing branches.



⇒ here R is input node or source.

(iv) output node or sink:

It is a node with only incoming branches. However this condition is not always met. An additional branch with unity gain may be introduced in order to meet the specified condition.



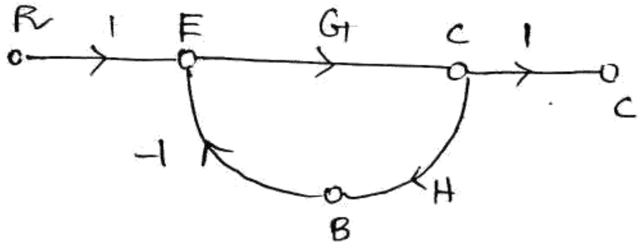
⇒ here c is output node or sink.

(v) path:

It is the traversal of connected branches in the direction of branch arrows such that no node is traversed more than once.

vi) <u>Forward path:</u>

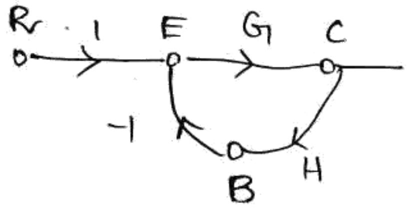
It is a path from input node to output node when no node encountered twice.



⇒ here R-E-C is a forward path.

vii) <u>Forward path gain :-</u>

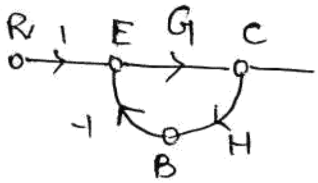
It is a product of branch gains in forward path.



⇒ here 'G' is forward path gain

viii) <u>Loop:</u>

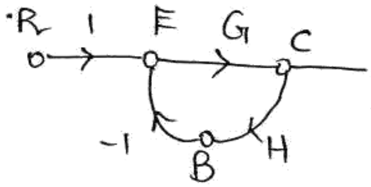
It is a path which originates and terminates at the same node.



⇒ here ECBE is a loop.

ix) <u>Loop gain:-</u>

It is a product of branch gains encountered in traversing the loop.



⇒ here "-GH" is loop gain for the loop E-C-B-E.

x) <u>Non-touching loop :-</u>

If they donot possess any common node. It is called non-touching loop.

Mason's Gain Formula :-

According to Mason's gain formula, the overall gain 'T' is expressed as,

$$\boxed{T = \frac{1}{\Delta}\sum_{K=1}^{N} P_K \Delta_K} = \frac{X_{out}}{X_{in}}.$$

where P_K → path gain of K^{th} forward path.

Δ = determinant of the path.

= 1 − (sum of loop gains of all individual loop) + (sum of gain products of all possible combinations of two non touching loops) − (sum of gain products of all possible combinations of three non touching loops) + ...

$$\boxed{\Delta = 1 - \sum_m P_{m1} + \sum_m P_{m2} - \sum_m P_{m3} + \cdots}$$

Δ_K = Value of Δ for that part of graph non touching the K^{th} forward path.

T = overall gain of system.

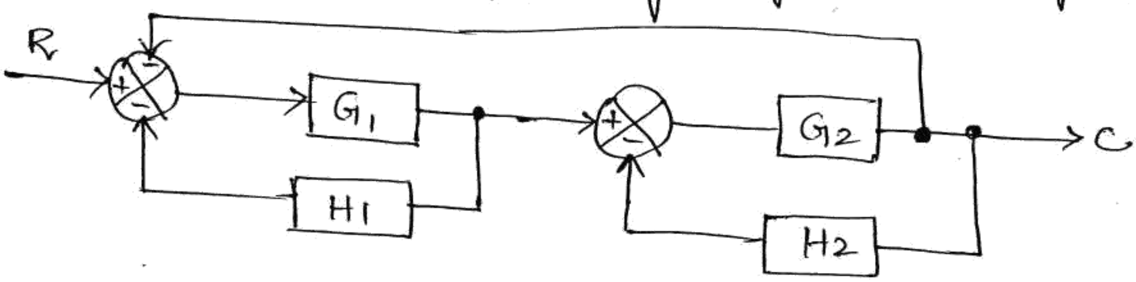
P_{mr} = gain product of m^{th} possible combination of 'r' non touching loops.

N = total number of forward paths.

— x — x —

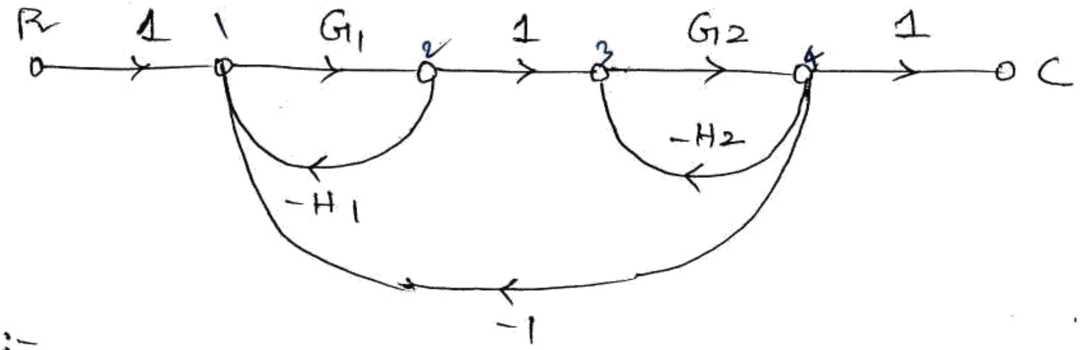
SFG problems are given by either equations or by Block diagrams or given as graph.

1. obtain the transfer function of a system given in Block diagram by using signal flow graph.



= Step 1:

The signal Flow graph for the system is drawn below,



Step 2:-

Finding out forward path.

here, Forward path is only one. i.e., R–C & its gain $= G_1G_2$

Step 3:

Finding out individual loops.

here, there are three individual loops, gains are

(i) $P_{11} = -G_1H_1$

(ii) $P_{12} = -H_2G_2$

(iii) $P_{13} = -G_1G_2$.

<u>Step 4:</u>

Finding out two non touching loops with gain.

$$P_{12} = (-G_1H_1)(-G_2H_2) = G_1G_2H_1H_2.$$

<u>Step 5:</u>

There is one forward path and all loops touch the forward path. $\therefore \boxed{\Delta_1 = 1}$

<u>Step 6:</u> Finding out 'Δ'.

$\Delta = 1 -$ (Sum of loop gain of all indi. loops) + (Sum of gain product of all possible combination of two non touching) $- \ldots$

$$\boxed{\Delta = 1 - (-G_1H_1 - G_2H_2 - G_1G_2) + G_1H_1G_2H_2}$$

<u>Step 7:-</u> Mason's gain Formula: $\boxed{T = \frac{1}{\Delta} \sum_K P_K \Delta_K}$

$$T = \frac{P_1\Delta_1}{\Delta} = \frac{G_1G_2}{1 + G_1H_1 + G_2H_2 + G_1G_2 + G_1G_2H_1H_2}$$

$(\because k=1)$

$$\boxed{T = \frac{G_1G_2}{(1+G_1H_1)(1+G_2H_2) + G_1G_2}}$$

—— x —— x ——

2. Obtain the transfer functions of the following set of linear equation by using signal flow graph.

$$x_2 = t_{12}x_1 + t_{32}x_3$$
$$x_3 = t_{23}x_2 + t_{43}x_4$$
$$x_4 = t_{24}x_2 + t_{34}x_3 + t_{44}x_4$$
$$x_5 = t_{25}x_2 + t_{45}x_4.$$

Solution:

Here input node is x_1 and output node is x_5.

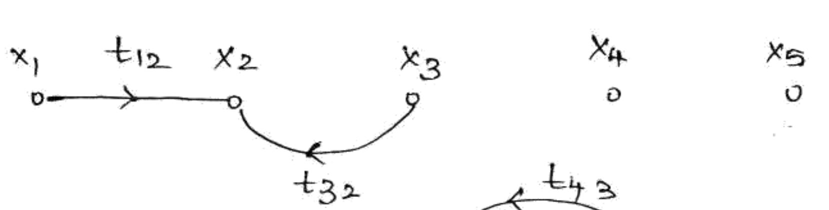
Step 1:-

locate the nodes of the system. Totally 5 nodes. and it can be represented by,

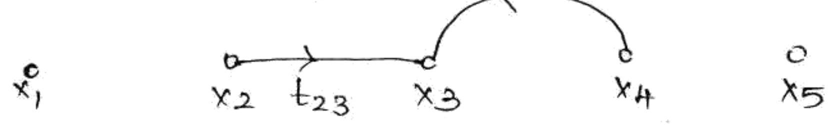
o	o	o	o	o
x_1	x_2	x_3	x_4	x_5

Step 2:-

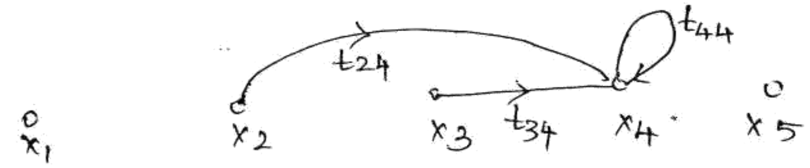
Draw SFG for four equations :

(i)


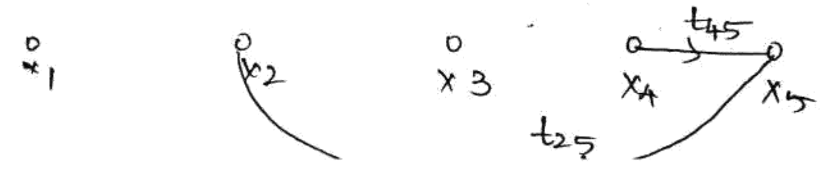
$\longrightarrow$ eq (1)

(ii)


$\longrightarrow$ eq (2)

(iii)


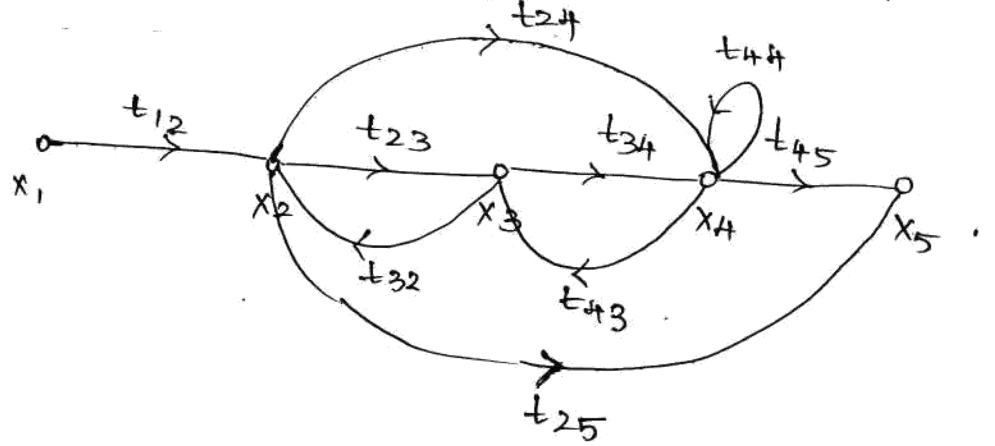
$\longrightarrow$ eq (3)

(iv)


$\longrightarrow$ eq (4)

Step 3 :-

Over all signal flow graph is obtained by adding the graphs of individual equations.



Step 4 :- Finding out forward path gain.

$P_1 = t_{12}\, t_{23}\, t_{34}\, t_{45}$

$P_2 = t_{12}\, t_{24}\, t_{45}$

$P_3 = t_{12}\, t_{25}.$

Step 5 : Finding out individual feedback loops.

$P_{11} = t_{23}\, t_{32}$

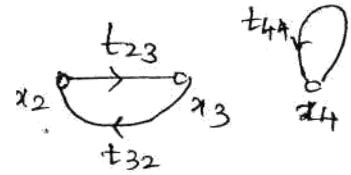
$P_{21} = t_{34}\, t_{43}$

$P_{31} = t_{44}$

$P_{41} = t_{32}\, t_{43}\, t_{24}$

Step 6 : There is only one possible combination of two non touching loops.

$P_{12} = t_{23}\, t_{32}\, t_{44}$



Step 7:-

The first forward path touches all loops, so no individual loop is formed $\therefore \boxed{\Delta_1 = 1}$

The second forward path is eliminated, no individual loop is formed. $\therefore \boxed{\Delta_2 = 2}$

The third forward path is eliminated, two loops are formed.

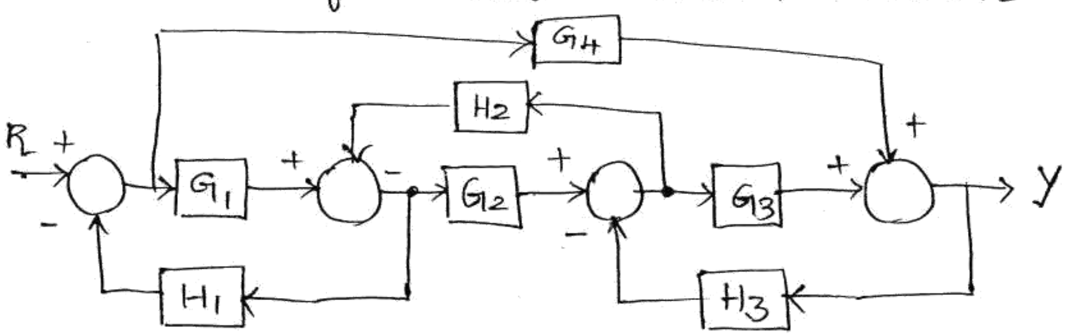
$$\therefore \boxed{\Delta_3 = 1 - t_{34}t_{43} - t_{44}}$$

Step 8:- Mason's gain formula:-

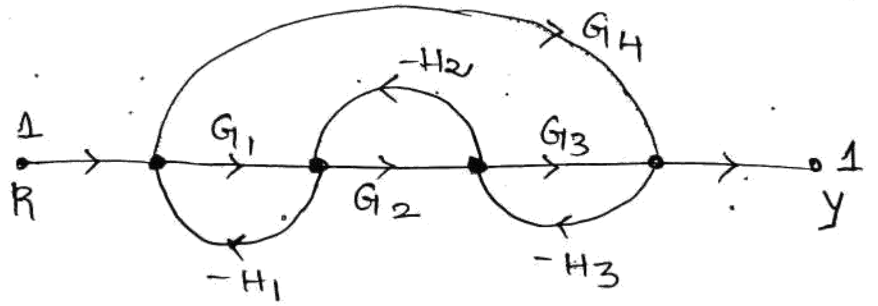
$$T.F = \frac{x_5}{x_1} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$$

$$T.F = \frac{(t_{12}t_{23}t_{34}t_{45}) + (t_{12}t_{24}t_{45}) + t_{12}t_{25}(1 - t_{34}t_{43} - t_{44})}{1 - (t_{23}t_{32} + t_{34}t_{43} + t_{44} + t_{32}t_{43}t_{24}) + t_{23}t_{32}t_{44}} \quad //.$$

3) Consider the block diagram as shown in Figure. Draw SFG & find out Transfer function.



Solution:- Negative feedback on block diagram are represented by assigning negative gains to feedback path on SFG.



⇒ Step ①

Step 2:- Finding out forward path with path gain.

$$\boxed{P_1 = G_1G_2G_3 \; ; \quad P_2 = G_4}$$

Step 3:- Finding out individual loops with loop gain

$P_{11} = -G_1H_1$; $P_{21} = -H_2G_2$; $P_{31} = -G_3H_3$ $P_{41} = -G_4H_3H_2H_1$

Step 4: Finding out loop gain product of two non touching loops.

$\boxed{P_{12} = G_1H_1G_3H_3}$ & there is no combination of more than two non touching loops.

$$\Delta = 1-(-G_1H_1 - G_2H_2 - G_3H_3 - G_4H_3H_2H_1) + G_1H_1G_3H_3$$

Step 5:- First forward path touches all loops

$\therefore \boxed{\Delta_1 = 1}$

Second forward path is not in touch with one loop. $\therefore \boxed{\Delta_2 = 1-(-G_2H_2)}$

Step 6:- Mason's Gain Formula: $T = \frac{1}{\Delta}\sum_K P_K\Delta_K$

$$= \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$

$$\boxed{T = \frac{G_1G_2G_3 + G_4 + G_4G_2H_2}{1+G_1H_1+G_2H_2+G_3H_3+G_4H_3H_2H_1+G_1H_1G_3H_3}}$$

//

DC & AC Servomotor

The word servomechanism originated from the words "servant (or slave) & Mechanism". The motors that are used in automatic control system are called "servomotors". When the objective of the system is to control the position of an object then the system is called "servomechanism".

electrical signal to mechanical signal

Servomotor — converts an electrical signal to angular velocity or movement of shaft.

Depends on supply req. to run motor

- **DC servomotor**
  - Sliding contact motor
    - Permanent Magnet motor
      - cylindrical armature motors
      - DISC armature motors
      - Moving coil motors
    - Electromagnetic field motor
      - Armature control motor
      - Field controlled motor
      - series motor
      - splitfield motor.
  - contact less motor.
- **AC Servomotor**

Requirements of Good servomotor :-

(i) It should be easily reversible, have linear torque-speed characteristics.

(ii) operation should be stable without any oscillation

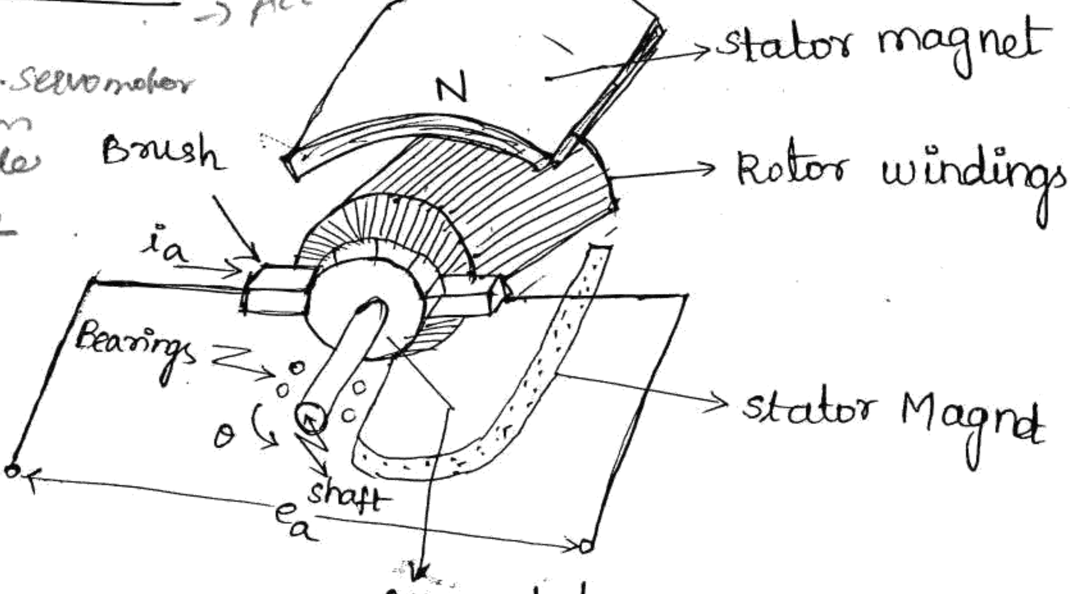
(iii) Linear relationship between speed and electric control signal.

(iv) low mechanical and electrical inertia.

(v) Fast response.

DC Servo motor :- → Same as DC motor.
→ Act as mechanical transducer which converts DC voltage into mechanical signal.

→ control of DC servomotor can be from
* Armature side
* Field side



The non turning part is called stator. It has magnets which establish a field across the turning part called rotor. The magnets may be electromagnets or for small motors, permanent magnet. In an electromagnet motor, stator is wound with wire and current is forced through this winding called field winding. For a constant field current the magnetic flux 'ϕ' is constant, ϕ may be varied by varying the field current.

The rotor is wound with wire and through this winding called armature winding, a current is forced through the stationary brushes and rotating commutator.

A DC servo motor is a specially designed DC motor with high starting torque and low inertia. A servo mechanism also called position control system, is a feedback control system and consists of a mechanism in which system may be some mechanical output of the position, velocity or acceleration.

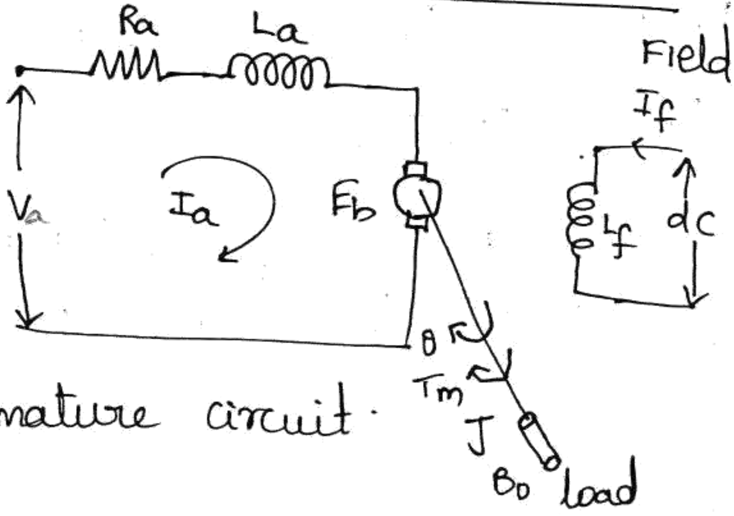
DC motor are used in two different control modes.

a) Armature control mode with constant field current and

b) Field control mode with fixed armature current.

a) Armature controlled DC motor :- → * Armature current is varied.
* Field current is held constant

Field circuit



Armature circuit.

* Ia → armature current
Eb → back emf
J → moment of inertia
Bo → dashpot
Lf → Field coil
Tm → Torque
Ra → armature Resistance
Va → armature voltage
ωm → Angular velocity

. In this method, speed is varied by changing the armature voltage keeping the field flux constant. The motor has rotating armature with load on its shaft having a moment of inertia J and viscous friction coefficient Bo. Torque developed by motor is T_m which causes rotation. The angular velocity is ω and angular acceleration is α. Torque of motor is due to armature current I_a and field flux ϕ. again ϕ is proportional to I_f.

$$T_m = K_1 \phi I_a$$

$\phi \propto i_f$
$\phi = K_f i_f$

$$T_m = K_1 K_f I_f I_a$$

$$\textcircled{1} \leftarrow \boxed{T_m = K_T I_a}$$ where $K_T = K_1 K_f I_f$

Back emf, $E_b = K\phi N = \frac{K\phi\omega}{2\pi} = \frac{KK_1 I_f}{2\pi}\frac{d\theta}{dt} = K_b \frac{d\theta}{dt} \rightarrow \textcircled{2}$ (ω → angular velocity) $\omega = \frac{d\theta}{dt}$

Inertia torque = Moment of inertia × Angular Acceler.

$$T_i = J \cdot \frac{d^2\theta}{dt^2} \rightarrow \textcircled{3}$$

Friction torque = Viscous Friction coeff × Ang. velocity

$$T_f = B_0 \omega = B_0 \frac{d\theta}{dt} \rightarrow \textcircled{4}$$

Torque developed by motor is opposed by inertia torque and frictional torque. Thus from Newtons II law of rotational motion,

$$T_i + T_f = T_m$$ ← Torque-load equation

$$J \cdot \frac{d^2\theta}{dt^2} + B_0 \frac{d\theta}{dt} = T_m = K_T I_a \rightarrow \textcircled{5}$$

The differential equation involving quantities of armature circuit can be written as

$$L_a \cdot \frac{dI_a}{dt} + R_a I_a + E_b = V \rightarrow \textcircled{6}$$

Taking Laplace transform of equation (2), (5) & (6).

$$E_b(s) = K_b s\,\theta(s) \longrightarrow (7)$$

$$(Js^2 + B_0 s)\,\theta(s) = K_T I_a(s) = T_m(s) \longrightarrow (8)$$

$$(L_a s + R_a) I_a(s) = V(s) - E_b(s) \longrightarrow (9)$$

From eqn (8) & (9)

$$[Js^2 + B_0 s]\,\theta(s) = K_T \frac{V(s) - E_b(s)}{L_a s + R_a}.$$

using eqn (2) in the above,

$$[Js^2 + B_0 s][L_a s + R_a]\,\theta(s) = K_T V(s) - K_b K_T s\,\theta(s)$$

$$\left[(Js^2 + B_0 s)(L_a s + R_a) + K_T K_b s\right]\theta(s) = K_T V(s).$$

$$\text{Transfer function} = \frac{\theta(s)}{V(s)} = \frac{K_T}{(Js^2 + B_0 s)(L_a s + R_a) + K_T K_b s}$$

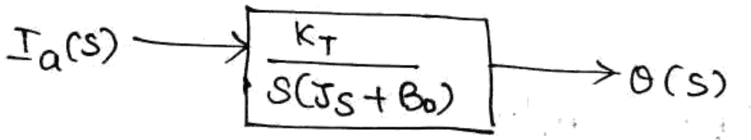
$$T.F = \frac{V_T}{s\left[(Js + B_0)(L_a s + R_a) + K_T K_b\right]}$$

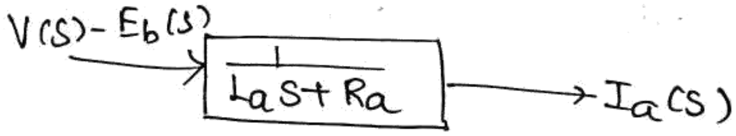
<u>Block Diagram Representation,</u>

From eq (9), we have $\dfrac{I_a(s)}{V(s) - E_b(s)} = \dfrac{1}{L_a s + R_a} \longrightarrow (10)$

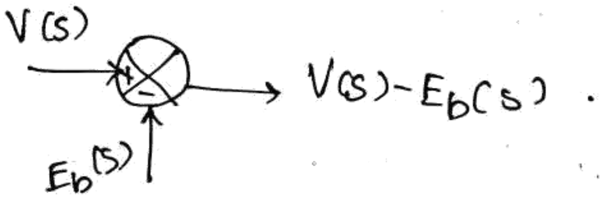
From eq (8), we have $\dfrac{\theta(s)}{I_a(s)} = \dfrac{K_T}{s(Js + B_0)} \longrightarrow (11)$

eq (10) & (11) can be represented by,



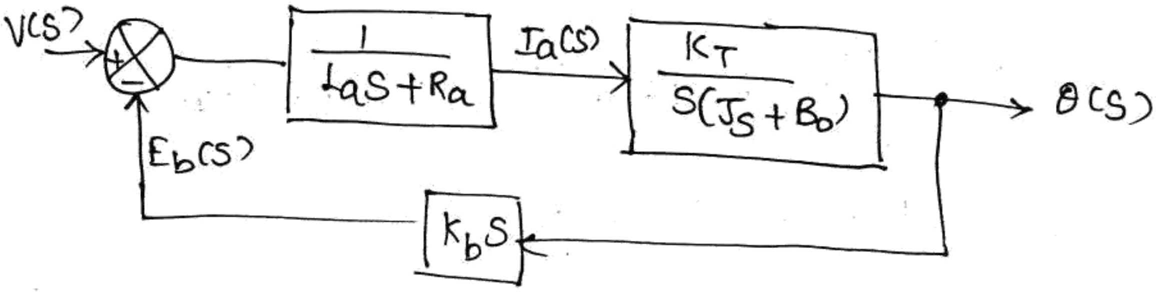


with $V(s)$ as input signal and $E_b(s)$ as feedback signal. the component is represented as,

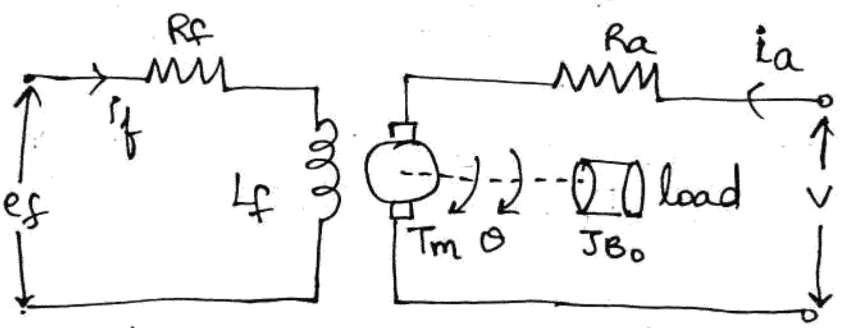


Aagain eq (7), $\boxed{E_b(s) = K_b s\theta(s)}$

combining all the above blocks, it will become.



ii) Field controlled DC Motor:-



field circuit

Armature circuit

$$T_m = K_1 \phi i_a = K_1 K_f i_f i_a = K i_f.$$

$$\boxed{K = K_1 K_f i_a}$$

Also field circuit equation,

$$e_f = L_f \frac{di_f}{dt} + R_f i_f \longrightarrow ①$$

Torque equation is,

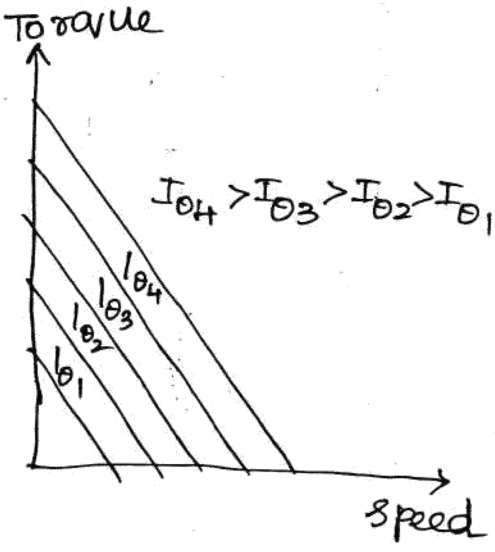
$$J\frac{d^2\theta}{dt^2} + B_0 \frac{d\theta}{dt} = T_m = K i_f \longrightarrow ②$$

Taking Laplace transform on above eqns.

$$(L_f S + R_f) I_f(S) = E_f(S) \longrightarrow ③$$

$$(JS^2 + B_0 S)\,\theta(S) = K I_f(S). \longrightarrow ④$$

$$T.F = \frac{\theta(S)}{E_f(S)} = \frac{K}{S(JS + B_0)(L_f S + R_f)}$$



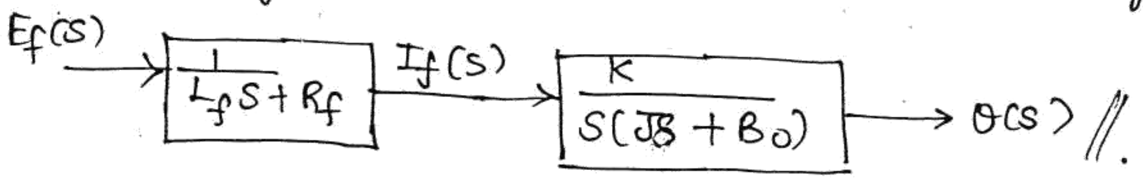
From eq ③,

$$\frac{I_f(S)}{E_f(S)} = \frac{1}{L_f S + R_f}$$

From eq ④,

$$\frac{\theta(S)}{I_f(S)} = \frac{K}{JS^2 + B_0 S}.$$

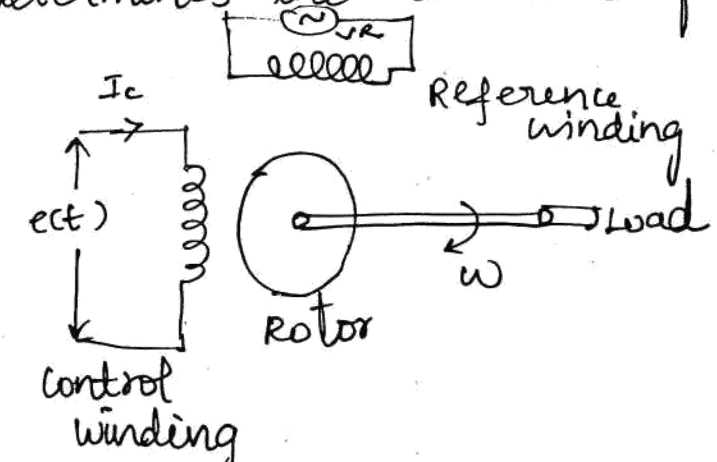
combining all the eqns in block diagram,

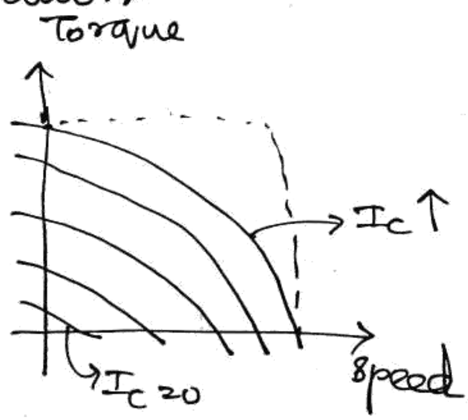


———×———×———

AC SERVO MOTOR:-

AC servo motors are two phase induction motor suitable for simple and low power applications. It has stator having two field windings placed at right angles to each other in order to produce a rotating field on which motor action depends. One phase is supplied from constant AC reference voltage V_R, the other phase act as a controlling field and is supplied from output of servo amplifier. The speed of rotation is proportional to control current I_c, the phase of which determines the direction of rotation.





It can be seen that curve for zero control current goes through origin and the slope is negative when control current becomes zero, the motor develops a deceleration torque, causing it to stop. The curve also shows a large torque at zero speed at increased I_c. For normal induction motor the ratio of X_2/R_2 is high. But for servo motors, X_2/R_2 ratio is kept low to obtain linear torque-speed characteristics. High value of R_2 will ensure high starting torque.

$$T_M = f(\theta, E)$$

$$T_M = T_{M0} + k(E - E_0) - f(\dot{\theta} - \dot{\theta}_0) \rightarrow (1)$$

where $k = \left.\frac{\delta T_m}{\partial E}\right|_{E = E_0, \dot{\theta} = \theta_0}$

$$f = \left.\frac{-\delta T_m}{\partial \dot{\theta}}\right|_{E = E_0,\ \dot{\theta} = \dot{\theta}_0}$$

from (1), $T_M - T_{M0} = k(E - E_0) - f(\dot{\theta} - \dot{\theta}_0)$

$$\Delta T_M = k\Delta_E - f\Delta\dot{\theta}$$

where $(\Delta T_M = T_M - T_{M0})$ and so on.

If J and f_0 be the inertia and viscous friction coefficient respectively of the load, the torque equation becomes,

$$\Delta T_M = J\Delta\dot{\theta} + f_0 \Delta\dot{\theta} = k\Delta E - f\Delta\dot{\theta}$$

Taking Laplace Transform,

$$(Js^2 + f_0 s)\Delta\theta(s) = k \, \Delta E(s) - fs\Delta\theta(s).$$

when an AC motor is used in position control system, operating point becomes $E_0 = 0$, $\dot{\theta}_0 = 0$, $\Delta\theta = \theta$, $\Delta E = E$.

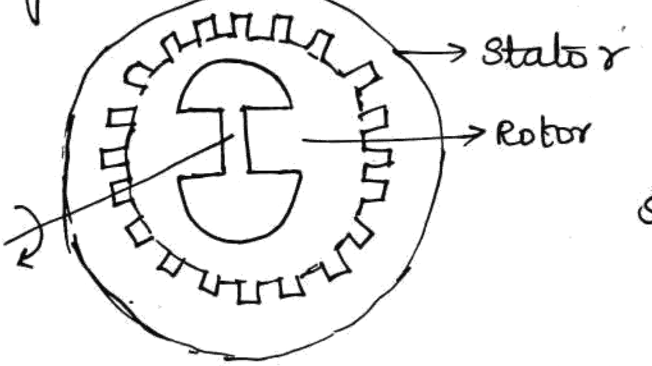
$$G(s) = \frac{\theta(s)}{E(s)} = \frac{K_m}{s(\tau_m s + 1)}$$

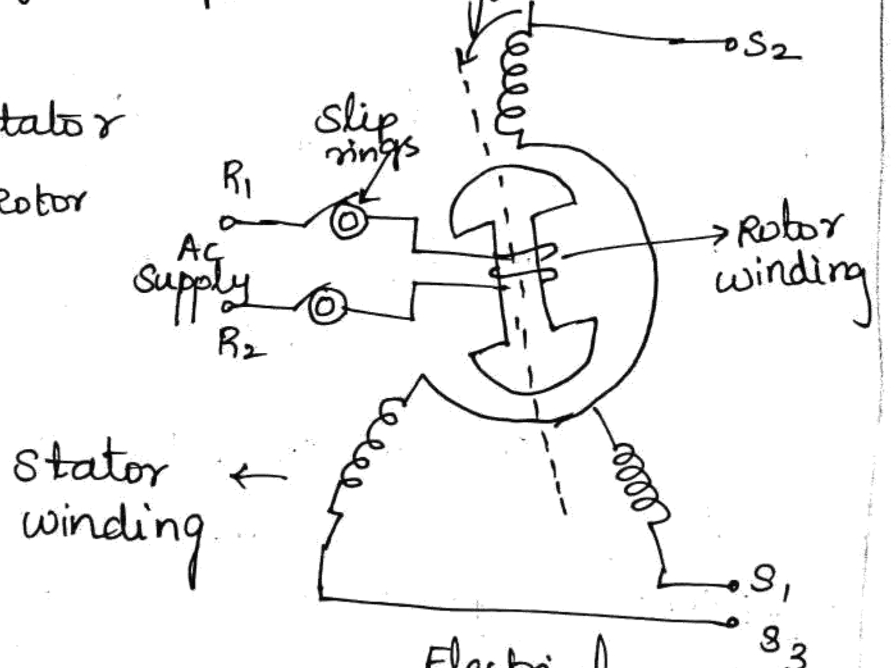
where $K_m = \frac{K}{f_0 + f}$; $\tau_m = \frac{J}{f_0 + f}$.

motor gain, motor time constant.

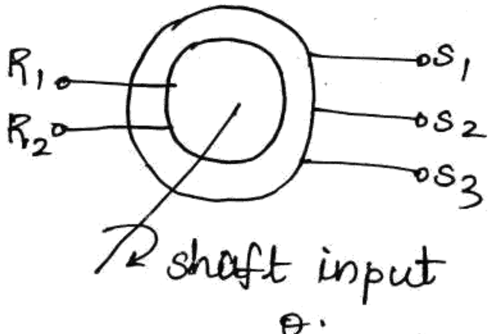
Synchros

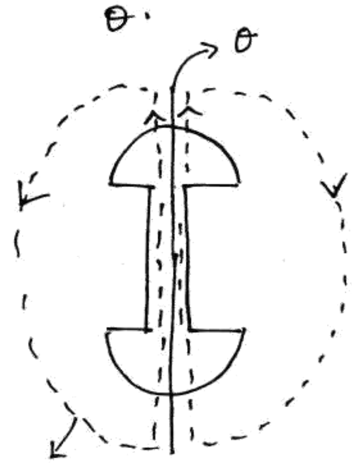
A synchro system formed by interconnection of the devices called synchro transmitter and the synchro control transformer. It is most widely used for error detector in feedback control systems. It measures and compares two angular displacements and its output voltage is approximately linear with angular difference.



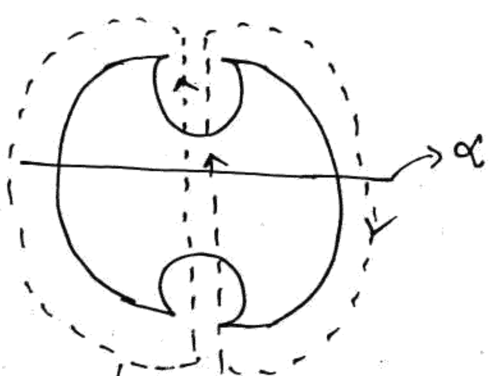


Electrical circuit





Flux Pattern.



flux pattern.

The stationary part of machine (stator) is slotted to accomodate three Y connected coils wound with their axes 120° apart. The stator windings are not directly connected to the ac power source. Their excitation is supplied by ac magnetic field produced by the rotor.

The rotor of a dumb-bell construction with a single winding. A single phase excitation voltage is applied to the rotor through two slip rings.

The resultant current produces a magnetic field and by the transformer action, induces voltages in the stator coils. The effective voltage induced in any stator coil depends upon angular position of coil axis with respect to the rotor axis

It offer higher sensitivity, longer life, ruggedness and continous rotation capability.

Since three stator windings of synchro transmitter are connected respectively in parallel with three stator windings of control transformer, stator winding induced voltage of synchro transmitter would cause current to flow and produce a resultant flux in the air-gap of synchro control transformer, which in turn will induce an emf in rotor. The magnitude of induced emf would be proportional to θ_r and θ_0. If difference $(\theta_r - \theta_0) = 0$, emf induced will be zero.

At 90°, the induced emf will be maximum.
Infact, the magnitude of induced emf at o/p, ie., across rotor terminals, will have relation,

$$e = K_s \sin(\theta_r - \theta_o)$$

$$e = K_s(\theta_r - \theta_o)$$

$$\boxed{e = K_s \theta_e}$$

($\because \sin\theta \simeq \theta$ if θ is small)

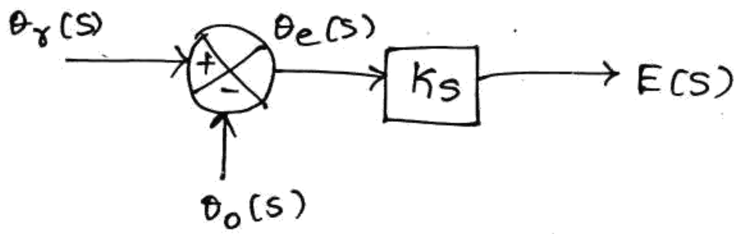
$K_s \rightarrow$ Proportionality Constant.

Taking Laplace Transform,

$$E(s) = K_s[\theta_r(s) - \theta_o(s)]$$

$$\boxed{E(s) = K_s \, \theta_e(s)}$$

Block Diagram :-



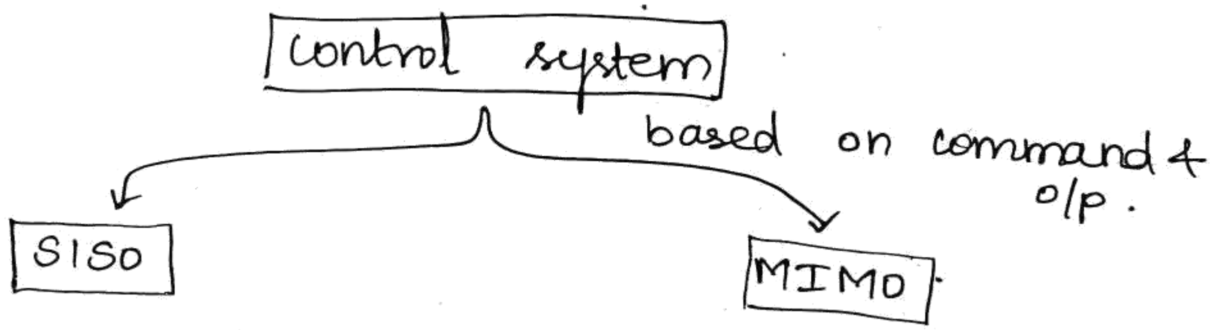
$$\boxed{K_s = \frac{E(s)}{\theta_e(s)} = \frac{E(s)}{\theta_r(s) - \theta_o(s)}}$$

where K_s is known as the sensitivity or the gain of error detector.

Synchros are used widely in control systems as detectors and encoders because their rigidness in construction and high reliability.

— x — x —

Multivariable control systems



SISO:

one input affect primarily one output and has only weak effect on other outputs. It is possible to ignore weak interactions (coupling) and design controllers under the assumption that one input affect only one output.

MIMO:

It consists of appropriate number of separate SISO systems. coupling effects are considered as disturbance to separate control systems and may not cause significant degradation in their performance if the coupling is weak.

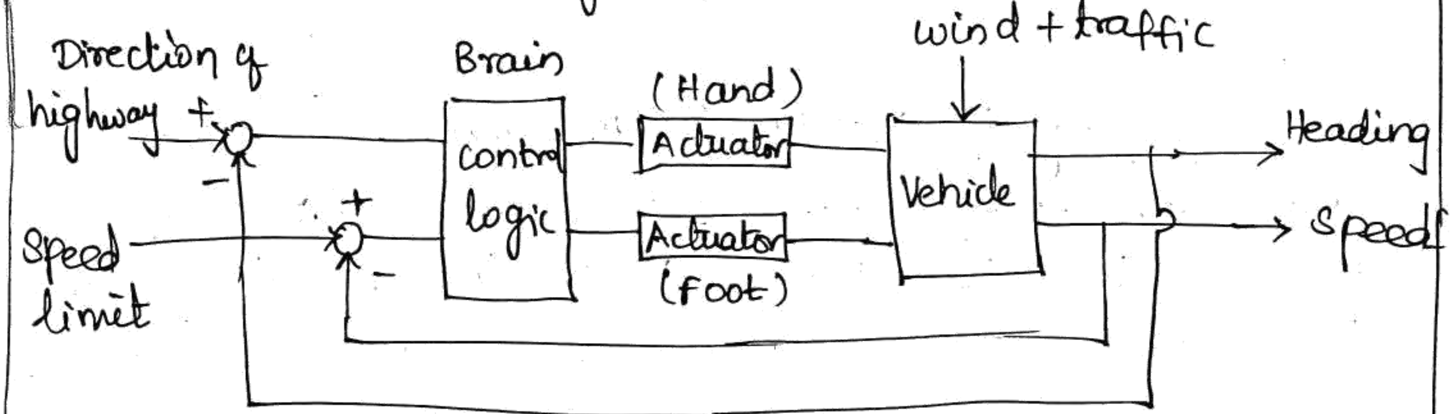
If it have strong interation (coupling) if one input affect more than one output appreciably. There are two approaches for design of controllers for each system.

(i) Design a decoupling controller to cancel the interaction inherent in the system.

(ii) Design a single controller for multivariable system. taking interaction into account.

Examples of Multivariable control system are
(i) Automobile Driving System.
(ii) Antenna stabilization system.
It can be illustrated as follows.

(i) Automobile Driving System:



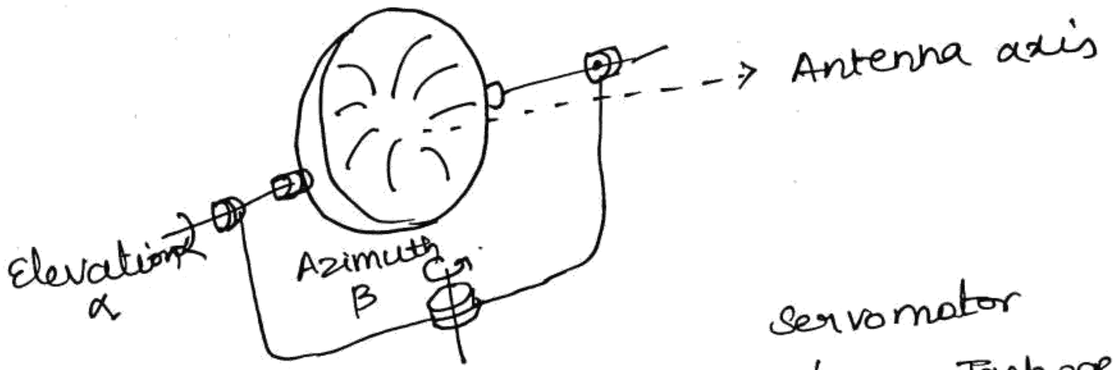
The system to be controlled has two inputs (steering & Acceleration/braking) and has two controlled outputs (heading & speed). Automobile driving system is a MIMO system. It can decouple this system in to two SISO system for the purpose of design.

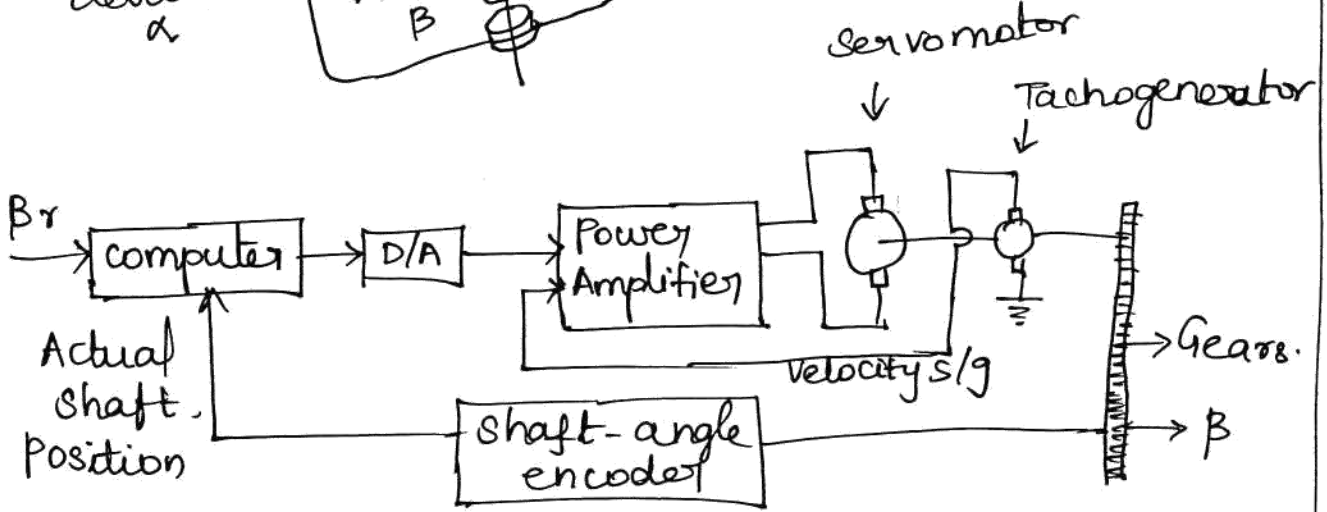
Steering control affects the heading and not speed and the accelerator control affect the speed and not heading. However brake of vehicle for speed control decreases the side forces at the tyre-road interface for directional control and with locked wheels the directional control is completely lost.

The command inputs cannot be constant set points. These inputs depends on traffic and road conditions and vary in an uncontrolled manner.

The actual signal points to the system are derived by driver from actual road and traffic conditions. The human operator ~~subsystem~~ subsystem will therefore be a component in the overall control system of the automobile. human operator then controls the manipulated variables in a manner which reduces absolute error.

(ii) Servomechanism for Steering of Antenna:-





The control system has for steering of an antenna can be treated as two independent s/m.

a) Azimuth angle servomechanism.

b) elevation angle servomechanism.

This is because interaction affect are usually small.

The occurence of azimuth angle error angle is ~~comp~~ causes an error signal to pass through amplifier, which increases the angular velocity of the servomotor in a direction towards an error reduction.

The main disturbance input is the deviation of load from the nominal estimated value as a result of uncertainty in our estimate, effect of wind power etc.,

Here windpower is disturbance in controlled system. Target surface fluctuations is a measurement noise.

These are the examples for Multi-Variable control systems.

— x — x —