Unit 5 - State variable analysis.

Introduction

- Analysis & design of linear/nonlinear Time variant /Invariant, MIMO system was carried using state space method.

→ State space analysis is a modern approach & easier for analysis using digital computers.

- Conventional mtd of analysis is tr fn of system. Drawback of tr fn

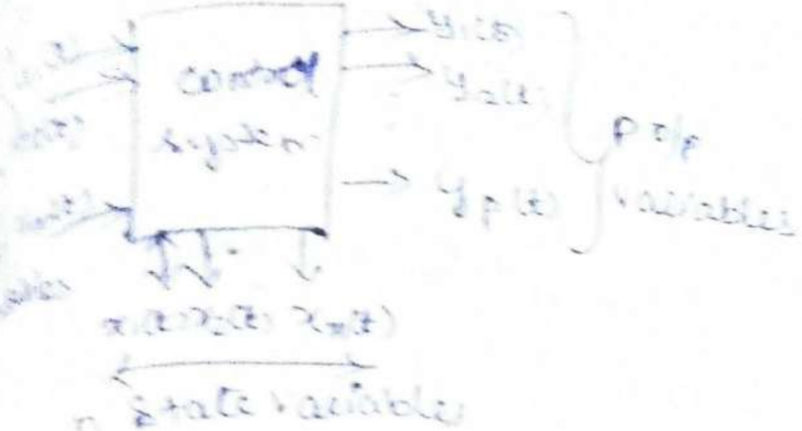
1) It is defined under initial condns
2) It is applicable to linear time invariant sys
3) It is restricted to SISO systems
4) Doesn't provide inf reg Internal state of state

Adv of State variable analysis

1) Applied for any type of sys
2) Analysis can be carried with initial condns & can be carried on MIMO sys.

State variable

- A set of variables which describe the sys at any time instant.



State space rep of sys

i/p vector $u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}$ o/p vector $y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix}$

State variable vector $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$

Diff eqn rep

$$\dot{X}(t) = \frac{dX}{dt} = f\left[X(t), u(t)\right]$$

State model of Linear system

- consists of state eqn & o/p eqn
- State eqn of a sys is a fn of state variables & i/ps. $\dot{X}(t) = f(X(t), u(t))$

for LTI system 1st derivative of state variables can be expressed as linear combination of state variables & i/ps

$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + b_{11}u_1 + b_{12}u_2 + \dots + b_{1m}u_m$

$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n + b_{21}u_1 + b_{22}u_2 + \dots + b_{2m}u_m$

$\vdots$

$\dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n + b_{n1}u_n + b_{n2}u_2 + \dots + b_{nm}u_m$

a, b consts

matrix rep

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \underbrace{\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ \vdots & \vdots & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}}_{\text{order}(n \times n)} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}}_{n \times 1} + \underbrace{\begin{bmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ b_{31} & b_{32} & \dots & b_{3m} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nm} \end{bmatrix}}_{n \times m} \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_m \end{bmatrix}}_{m \times 1}$$

matrix eqn of LTI system $\boxed{\dot{X}(t) = A\,X(t) + B\,U(t)} \rightarrow$ state eqn

o/p at any time is fn of state variables & i/ps

o/p vector $\boxed{Y(t) = f(X(t), U(t))}$

$y_1 = c_{11}x_1 + c_{12}x_2 + \dots + c_{1n}x_n + d_{11}u_1 + d_{12}u_2 + \dots + d_{1m}u_m$

$y_2 = c_{21}x_1 + c_{22}x_2 + \dots + c_{2n}x_n + d_{21}u_1 + d_{22}u_2 + \dots + d_{2m}u_m$

$\vdots$

$y_p = c_{p1}x_1 + c_{p2}x_2 + \dots + c_{pn}x_n + d_{p1}u_1 + d_{p2}u_2 + \dots + d_{pm}u_m$

c, d → consts

$$\boxed{\underset{p \times 1}{Y(t)} = \underset{p \times n}{C}\ \underset{n \times 1}{X(t)} + \underset{p \times m}{D}\ \underset{m \times 1}{U(t)}} \rightarrow \text{o/p eqn of LTI sys}$$

order ⇒

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & & & \vdots \\ c_{p1} & c_{p2} & \cdots & c_{pn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1m} \\ d_{21} & d_{22} & \cdots & d_{2m} \\ \vdots & & & \vdots \\ d_{p1} & d_{p2} & \cdots & d_{pm} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_m \end{bmatrix}$$

State diag

- pictorial rep of state model of sys is called state diagram.
- State diag of sys can be either block diag or in signal flow graph form.
- State diag describes the relationship b/w/among state variables & provides physical interpretation of state variables.
- Time domain state diag is obtained from diff eqn governing system & this diag can be used for simulation of sys in analog comp.
- s domain state diag can be obtained from tr. Fn of system.
- State diag provides direct relationship b/w time & s-domain.
- State diagram of a state model is constructed using 3 basic elements

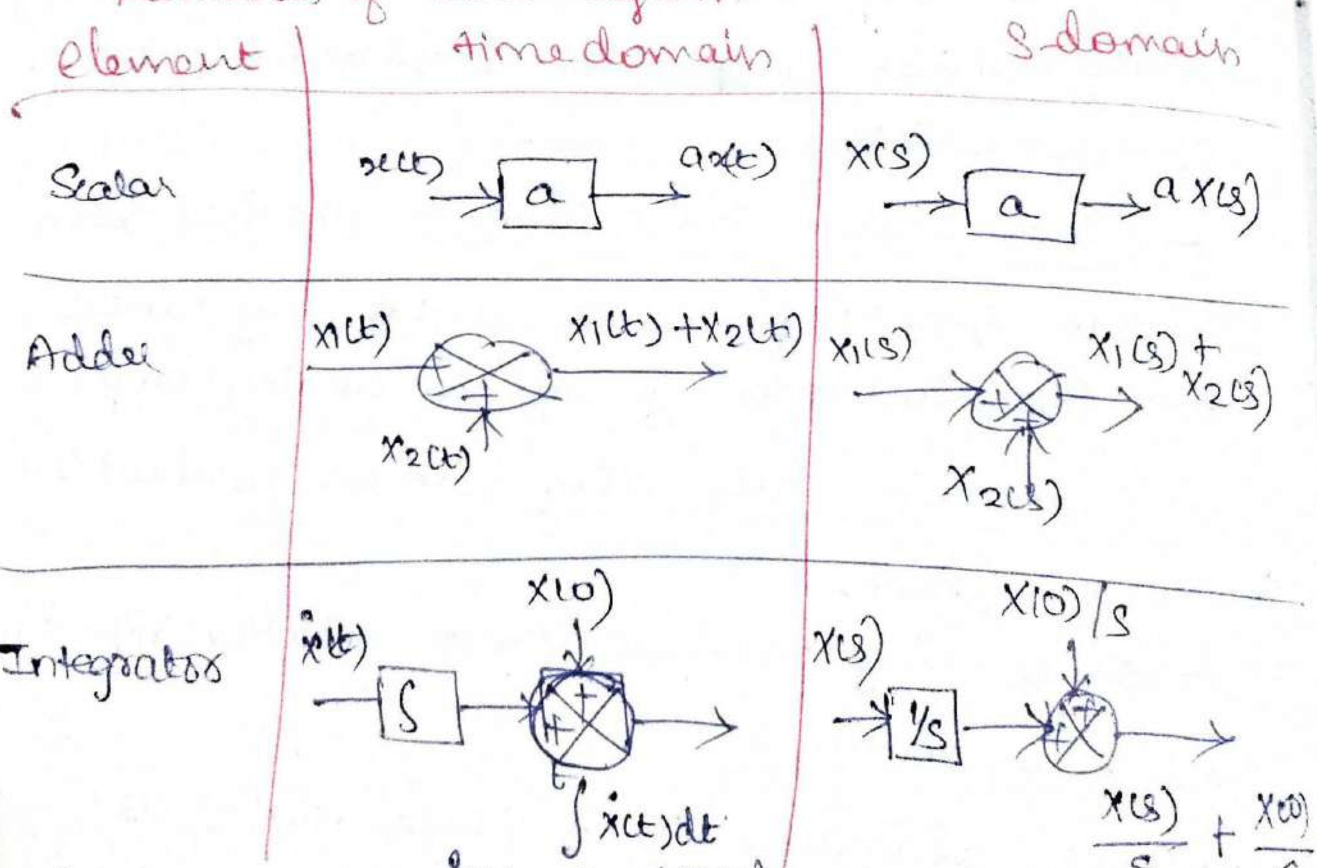
Scalar, adder & Integrator

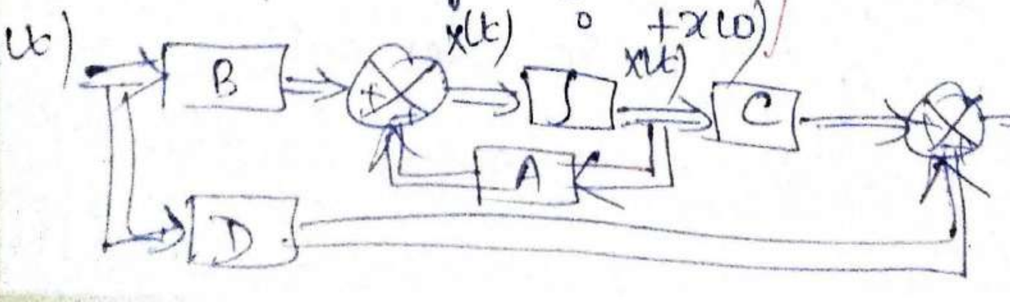
Scalar - used to multiply a signal by a const. I/p $x(t)$ is multiplied by a scalar 'a' to give o/p $(x(t) \cdot a)$

Adder: Adds 2 or more signals. o/p of adder is sum of incoming signals

Integrator: Integrates the signal. used to integrate the derivatives of state variables to get state variables. Initial condns of state variables can be added by using an adder after integrator.

Elements of blockdiagram



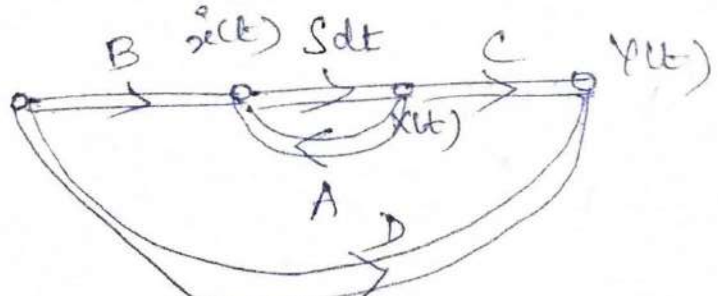


$\dot{x}(t) = Ax(t) + Bu$

$y(t) = Cx(t) + du$

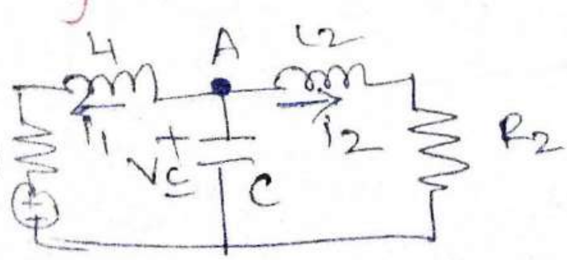
Elements of signal flow graph

Element	time domain	s-domain
Scalar	$x(t) \xrightarrow{a} a x(t)$	$X(s) \xrightarrow{a} aX(s)$
Adder	$x_1(t) \xrightarrow{1} x_1(t)+x_2(t)$; $x_2(t) \xrightarrow{1} x_1(t)+x_2(t)$	$X_1(s) \xrightarrow{1} X_1(s)+X_2(s)$; $X_2(s) \xrightarrow{1} X_1(s)+X_2(s)$
Integrator	$\dot{x}(t) \xrightarrow{\int dt} x(t)+x(0)$; $x(0) \xrightarrow{1} x(t)+x(0)$	$X(s) \xrightarrow{1/s} \frac{X(s)}{s}+\frac{x(0)}{S}$; $x(0)/S \xrightarrow{1} \frac{X(s)}{s}+\frac{x(0)}{S}$



problem

1) Obtain state model of electrical n/w by choosing min no. of state variables



$x_1 = i_1 = I$ thro' L_1

$x_2 = i_2 = I$ thro' L_2

$x_3 = V_C$ = voltage across C

At A, KCL, $\quad \dot{i}_1 + i_2 + C\frac{dV_C}{dt} = 0$

Sub state variables

$$x_1 + x_2 + C\dot{x}_3 = 0$$

$$C\dot{x}_3 = -x_1 - x_2$$

$$\boxed{\dot{x}_3 = -\frac{1}{C}x_1 - \frac{1}{C}x_2}$$

KVL at loop 1

$$e(t) + i_1R_1 + L_1\frac{di_1}{dt} = V_C$$

Sub state variables

$$e(t) + x_1R_1 + L_1\dot{x}_1 = x_3$$

$u(t) = e(t)$ i/p to sys,

$$u + x_1R_1 + L_1\dot{x}_1 = x_3$$

$$L_1\dot{x}_1 = x_3 - x_1R_1 - u$$

$$\boxed{\dot{x}_1 = -\frac{R_1}{L_1}x_1 + \frac{1}{L_1}x_3 - \frac{1}{L_1}u}$$

By KVL at loop 2,

$$V_C = L_2\frac{di_2}{dt} + i_2R_2,$$

$$x_3 = L_2\dot{x}_2 + x_2R_2$$

Homogeneous eqn

If A is a const matrix & i/p ctrl forces are zero then eqn takes form $\boxed{\dot{x}(t) = Ax(t)}$

This is homogeneous eqn.

- If i/p is zero, in such sys, driving forces is provided by initial condns of sys to produce o/p.

Eg. consider a RC ckt in which capacitor is initially charged to V volts. I is o/p. Now there is no i/p ctrl force ie ext voltage applie to sys. But initial voltage on capacitor drives the current thro' sys & capacitor starts discharging thro' R. Such a sys which works on initial condns w/o any i/p applied to it is called homogeneous sys.

Non homogeneous eqn

If A is a const matrix & matrix U(t) is non zero vector ie i/p ctrl forces are applied to sys then eqn takes normal form as,

$\boxed{\dot{x}(t) = Ax(t) + BU(t)}$ → Non homogeneous eq

- most practical sys req i/ps to drive them. Such sys are non homogeneous linear sys. The solution of state eqn is obtained by considerin basic mtd of finding soln of homogeneous eqn

Controllability & observability

$\dot{x} = Ax + Bu$, $y = Cx + Du$, $x(0) = x_0$

$\hat{x} = Qx$ (Q = transformation matrix)

$x = Q^{-1}\hat{x}$

$\dot{\hat{x}} = \Lambda \hat{x} + \hat{B}u$, $y = \hat{C}\hat{x} + \hat{D}u$

where

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_n \end{bmatrix}$$

- By doing the diagonalizing transformation, resulting tr fn b/w $u(s)$ & $y(s)$ will not be altered
- If $\hat{b}_n = 0$, then $x_n(t)$ is uncontrollable by the $u(t)$, since $x_n(t)$ is characterized by mode $e^{\lambda_n t}$ by the eqn $x_n(t) = e^{\lambda_n t} x_n(0)$

- The lack of controllability of the state $x_n(t)$ is reflect by a zero n^{th} row of $\hat{B}$ i.e. $\hat{b}_n$, which would cause a complete zero rows in the matrix controllability matrix :

$S(\hat{A}, \hat{b}) = [\hat{b} \ \hat{A}\hat{b} \ \hat{A}^2\hat{b} \ \hat{A}^3\hat{b} \ \ldots \ \hat{A}^{n-1}\hat{b}]$

$$= \begin{bmatrix} \hat{b}_1 & \lambda_1 \hat{b}_1 & \lambda_1^2 \hat{b}_1 & & \lambda_1^{n-1} \hat{b}_1 \\ \hat{b}_2 & \lambda_2 \hat{b}_2 & \lambda_2^2 \hat{b}_1 & & \lambda_2^{n-1} \hat{b}_2 \\ - & - & - & - & - \\ \hat{b}_n & \lambda_n \hat{b}_n & \lambda_n^2 \hat{b}_n & - & \lambda_n^{n-1} \hat{b}_n \end{bmatrix}$$

— $C(A,b)$ matrix with all non zero row has rank of N.

In fact $B = G^{-1} B$ or $B = GB$, Thus a non Singular $C(A,b)$ matrix Implies a non-singular matrix $C(A,b) = [B \ AB \ A^2B \ \ A^{n}B]$

Tr Fn from State variable rep

$$u(t) \rightarrow \boxed{S} \rightarrow y(t)$$

General form of LTI system

$$\frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_0 y(t)$$

$$= b_{m-1} \frac{d^{m-1} u}{dt^{m-1}} + \dots + b_0 u(t)$$

This model has the property

$u(t) = a_1 u_1(t) + a_2 u_2(t) \Rightarrow y(t) = a_1 y(t) + a_2 y^{2t}$

Satisfies general form.

- Assume sys is at rest prior to time $t_0 = 0$, & i/p $u(t)$ $(0 < t < \alpha)$ produces the o/p $y(t)$ $(0 < \dots$ can be rep by a tr fn in term of LT variables

$$y(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_0} u(s)$$

Then applying the same i/p shifted by any amt(q) of time produces same o/p shifted by same amt of of time.

$$y(s) = \left(\frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0} \right) e^{-as} u(s)$$

having all $b_i = 0$ $(i > 0)$, a state space rep arose out of a reduction to a sys of 1st order diff eqns.

$$y^{(n)} = f\{t, u(t), y, \dot{y}, \ddot{y}, \ldots y^{(n-1)}\};$$

with initial condns $y(0) = y_0$, $\dot{y}(0), \ldots y^{(n-1)}(0) = y^{(0)}_{n-1}$

consider a vector $x \in \mathbb{R}^n$ with $x_1 = y$, $x_2 = \dot{y}$, $x_3 = \ddot{y} \ldots$ $x_n = y^{(n-1)}$

$$\frac{d}{dt} X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ f\ t, u(t), y, \dot{y}, \ddot{y}, \ldots y^{(n-1)} \end{bmatrix}$$

n case of linear sys

$$\frac{d}{dt} X = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & \cdots & & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-1} \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} X$$

General form

$$\frac{d}{dt}X = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & \cdots & & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-1} \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 1 \end{bmatrix} u(t)$$

$$y(t) = b_0 \; b_1 \; \cdots \; b_m \; 0 \cdots 0 \; X$$

Abbreviation

$$\boxed{\dot{X} = AX + Bu, \quad y = CX + Du \; ; D = 0}$$

Controller canonical form of sys

State space rep for discrete time sys

The dynamics of a linear time shift invariant DT sys may be expressed in terms of state (plant) eqn & o/p (observation or measurement) eqn as folls.

$$x(k+1) = Ax(k) + Bu(k),$$

$$y(k) = Cx(k) + Du(k)$$

where $x(k)$ is an n dimensional state vector a time $= kT$.

$u(k) \rightarrow r$ dimensional ctrl i/p vector.

$y(k) =$ m dimensional o/p vector.

$$X(k) = [x_1(k), x_2(k), \ldots x_n(k)]^T$$

$$u(k) = [u_1(k), u_2(k), \ldots u_n(k)]^T$$

$$y(k) = [y_1(k), y_2(k), \ldots y_n(k)]^T$$

The parameters of A, an $n \times n$ (plant parameter) matrix, B an $n \times r$ ctrl i/p matrix & C an $m \times r$ o/p parameter matrix, D an $m \times r$ parametrix matrix. All are consts for an LTI sys.

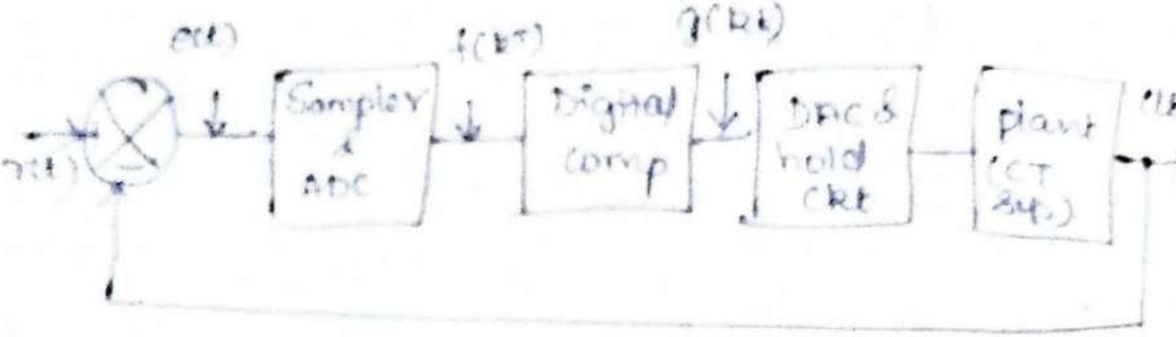
State variable rep of SISO DT sys

$$x(k+1) = Ax(k) + bu(k)$$
$$y(k) = c^T x(k) + du(k)$$

- where i/p u, o/p y & d are scalars, b & c are n dimensional vectors.
- The concepts of controllability & observability for DT sys are similar to continuous time sys.
- A DT sys is said to be controllable if there exists a finite integer n & i/p m u(k); k [0, n 1] that will transfer any state (o) $x^0 = bx(0)$ to state x^n at k = n.

Sampled Data sys

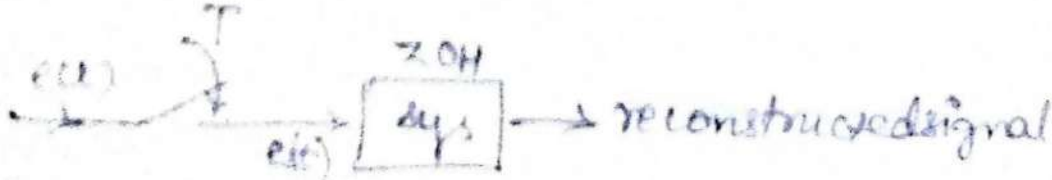
- When the signal or inf at any or s pts in a sys is in form of discrete pulses. Then sys is called discrete data sys. In ctrl sys engg the discrete data sys is popul known as sampled data sys



Sampling theorem

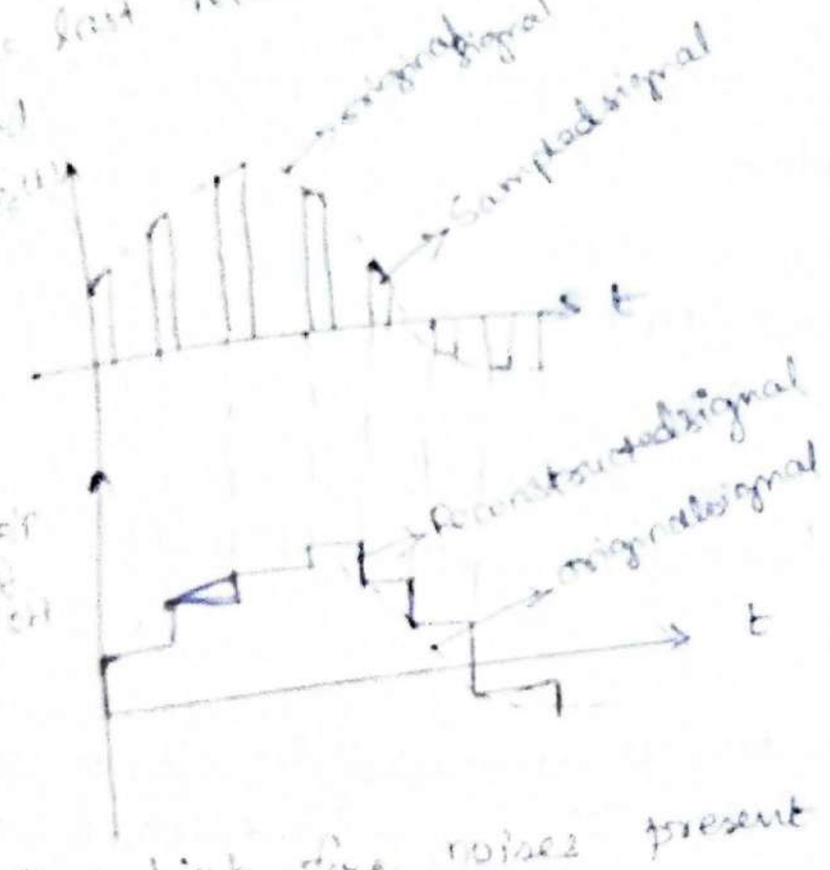
A Band lmted CT signal with highest freq (fm) Hz can be uniquely recovered from its samples provided that the sampling rate Fs is greater than or equal to 2fm samples per sec $\boxed{f_s \geq 2f_m}$

Sample & hold



- Signal gn to digital cotroller is a sampled data signal & in turn the controller gives the controller o/p in dig form. But the sys to be controlled needs an analog ctrl al as i/p ∴ the dig o/p of controllers must converted into analog form.

This can be achieved by means of various ... of hold ckts. The simplest hold ckts are ... order hold (ZOH). In ZOH, the reconstructed analog signal acquires the same values as the last received sample for entire sampling ...



- The high freq noises present in reconstructed signal are automatically filtered out by ctrl sys component which behaves like low pass filters. In 1st order hold, last 2 signals for current sampling period. 1st or higher order hold ckts have no particular adv over ZOH.

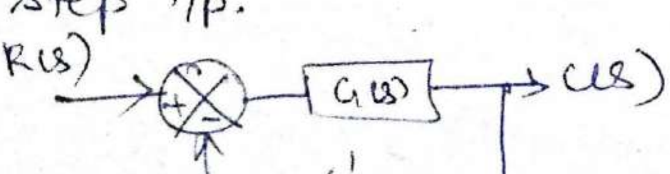
unit 2 matlab pgm

1) write matlab pgm to find unit step response of fol sys $M(s) = 4 / s^2+5s+4$

```
clc
syms s complex;
R = 1/s ;
m = 4 /(s^2 + 5*s +4) ;
S1 = R * M
disp ('unitstep response of system is');
s2 = ilaplace (s1)
t =0 : .005 : 10 ;
~~s3= step (m)~~
plot (t , s2);
xlabel ('time, t in sec');
ylabel ('unit step response s2(t)');
```

problem

1) unity FB system is characterised by an open loop tr fn $G(s) = \frac{K}{s(s+10)}$. Determine gain K, so that sys will have damping ratio of 0.5 for this value of K. Determine peak overshoot & time at peak overshoot for a unit step i/p.



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+10)}}{1+\frac{K}{s(s+10)}} = \frac{K}{s^2+10s+K}$$

compare tr fn with std form of 2nd order tr fn

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{K}{s^2+10s+K}$$

$\omega_n^2 = K$ $\quad 2\xi\omega_n = 10$ $\quad K = 100$

$\boxed{\omega_n = \sqrt{K}}$ $\quad \boxed{\xi = 0.5}$ $\quad \omega_n = 10\ \text{rad/sec}$

$\omega_n = \sqrt{K}$

$\sqrt{K} = 10$

$\therefore K = 100$

$$\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100 = 16.3\%$$

peak time $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 0.363$ sec.

) open loop tr fn of unity FB system is $G(s) = \frac{K}{s(sT+1)}$, where K & T are the constant. By what factor should the ampr gain K be reduced, so that peak overshoot of unit step response of sys is reduced from 75% to 25%.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{K/T}{s^2+\frac{1}{T}s+\frac{K}{T}} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\omega_n^2 = K/T$ $\quad 2\xi\omega_n = \frac{1}{T}$

$\omega_n = \sqrt{K/T}$ $\quad \xi = \frac{1}{2\omega_n T} = \frac{1}{2\sqrt{\frac{K}{T}}\cdot T} = \frac{1}{2\sqrt{KT}}$

When $M_p = 0.75$ let $\xi = \xi_1$ & $K = K_1$

$M_p = 0.25$ let $\xi = \xi_2$ & $K = K_2$

$$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

take ln,

$$\ln M_p = \frac{-\xi\pi}{\sqrt{1-\xi^2}}$$

Squaring

$$(\ln M_p)^2 = \frac{\xi^2\pi^2}{1-\xi^2}$$

$$(1-\xi^2)(\ln M_p)^2 = \xi^2\pi^2$$

$$(\ln M_p)^2 - \xi^2(\ln M_p)^2 = \xi^2\pi^2$$

$$(\ln M_p)^2 = \xi^2\left[\pi^2 + (\ln(m_p))^2\right]$$

$$\xi^2 = \frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2} \rightarrow (1)$$

$$\xi = \frac{1}{2\sqrt{KT}}, \quad \xi^2 = \frac{1}{4KT} \rightarrow (2)$$

$(1) = (2)$

$$\frac{1}{4KT} = \frac{(\ln m_p)^2}{\pi^2 + (\ln M_p)^2}$$

$$\frac{1}{K} = \frac{4T(\ln m_p)^2}{\pi^2 + (\ln M_p)^2}$$

$$K = \frac{\pi^2 + (\ln m_p)^2}{4T(\ln m_p)^2}$$

$K = K_1, m_p = 0.75)$ $$K_1 = \frac{\pi^2 + (\ln 0.75)^2}{4T(\ln 0.75)^2} = \frac{30.06}{T}$$

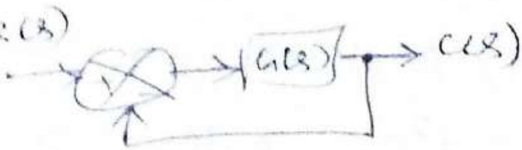
$M_p = 0.25$

$$\frac{\pi^2 + (\ln 0.25)^2}{4 + (\ln 0.25)^2} = \frac{1.53}{T}$$

$= 19.6 \approx 20$

$\therefore$ K should be reduced by 19.6 times

A unity FB ctrl sys has an open loop tr fn $G(s) = \frac{10}{s(s+2)}$. Find t_r, $\%M_p$, t_p & t_s for step i/p of 12 units



$H(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$= \frac{10}{s^2 + 2s + 10} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

comparing

$\omega_n^2 = 10$

$\boxed{\omega_n = \sqrt{10} = 3.162 \text{ rad/sec}}$

$2\xi\omega_n = 2$

$\boxed{\xi = \frac{2}{2\omega_n} = 0.316}$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) = 71.5^\circ \times \frac{\pi}{180} = 1.249 \text{ rad}$$

$\omega_d = \omega_n\sqrt{1-\xi^2} = 3 \text{ rad/sec}$

$t_r = \frac{\pi - \theta}{\omega_d} = 0.63 \text{ sec}$

$\%M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100 = 35.12\%$

Peak overshoot for step i/p of 12 units $= \frac{\% M_p}{100} \times 12$ units

$= \underline{4.2144 \text{ units}}$

for 5% error $T_s = 3T = \underline{3 \sec}$

2% error $T_s = 4T = \underline{4 \sec}$

$T = \frac{1}{\xi \omega_n} = \underline{1 \sec}$

$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3} = \underline{1.047 \sec}$

4) A closed loop servo is rep by diff eqn $\frac{d^2c}{dt^2} + 8\frac{dc}{dt} = 64e$, where c is the displacement of o/p shaft, r is displacement of i/p shaft & $e = r - c$. Determine undamped natural freq, daming ratio & % Mp for unit step i/p

put $e = r - c$

$$\frac{d^2c}{dt^2} + 8\frac{dc}{dt} = 64(r-c)$$

Take LT

$$s^2 C(s) + 8s\,C(s) = 64[R(s) - C(s)]$$

$$C(s)[s^2 + 8s + 64] = 64\,R(s)$$

$$\frac{C(s)}{R(s)} = \frac{64}{s^2+8s+64} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$\omega_n^2 = 64$ | $2\xi\omega_n = 8$ | $\% M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$

$\boxed{\omega_n = 8 \text{ rad/sec}}$ | $\boxed{\xi = 0.5}$ | $\boxed{\% M_p = 16.3\%}$

Consider a unity FB system with a closed loop T.F. $M(s) = \frac{Ks+b}{s^2+as+b}$. Determine open loop T.F. $G(s)$. Show that steady state error with unit ramp i/p is given by $\frac{(a-K)}{b}$

$H(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = M(s)$$

$$G(s) = M(s) + M(s)G(s)$$

$$G(s)(1 - M(s)) = M(s)$$

$$G(s) = \frac{M(s)}{1-M(s)} = \frac{\frac{Ks+b}{s^2+as+b}}{1 - \frac{Ks+b}{s^2+as+b}}$$

$$\boxed{G(s) = \frac{Ks+b}{s(s+a-K)}}$$

unit ramp i/p, $K_V = \lim_{s \to 0} s\, G(s)H(s)$

$$= \lim_{s \to 0} s \cdot \frac{Ks+b}{s(s+a-K)}$$

$$K_V = \frac{b}{a-K}$$

$$\boxed{e_{ss} = \frac{1}{K_V} = \frac{a-K}{b}}$$ Hence proved

b) A unity FB system has fwd to Fn $G(s) = \frac{K_1(2s+1)}{s(5s+1)(1+s)^2}$ i/p $r(t) = 1+6t$, determine K_1 so that SSE < 0.1

$$R(s) = L\{r(t)\} = \frac{1}{s} + \frac{6}{s^2}$$

$$E(s) = \frac{R(s)}{1+G(s)H(s)} = \frac{\frac{1}{s} + \frac{6}{s^2}}{1 + \frac{K_1(2s+1)}{s(5s+1)(1+s)^2}}$$

$$= \frac{1}{s}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right] + \frac{6}{s^2}\left[\frac{s(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right]$$

$$e_{ss} = \lim_{s \to 0} sE(s)$$

$$= \lim_{s \to 0} \cancel{s}\left[\frac{1}{\cancel{s}} \frac{s(5s+1)((1+s)^2)}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right] + \frac{6}{s^{\cancel{2}}} \cdot \cancel{s}\left[\frac{\cancel{s}(5s+1)(1+s)^2}{s(5s+1)(1+s)^2 + K_1(2s+1)}\right]$$

$$e_{ss} = 0 + \frac{6}{K_1} = \frac{6}{K_1}$$

$e_{ss} < 0.1$ $\therefore$ $0.1 = \frac{6}{K_1}$, $\boxed{K_1 = \frac{6}{0.1} = 60}$