UNIT: 1 SYSTEM COMPONENTS & THEIR REPRESENTATION

Control system: Terminologies & Basic structure - FeedBack & Feed Forward control theory - Electrical and Mechanical Transfer funtion models - Block diagram models - signal flow graph models - DC & AC servo motors - Synchros - Multivariable control systems.

— × —— × —— × ——

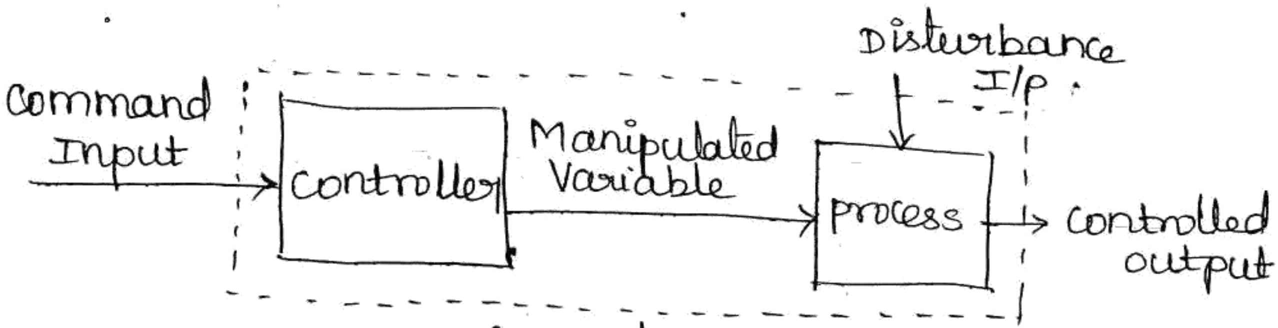
INTRODUCTION:

"Control systems" deal with control of Engineering systems that are governed by the "laws of physics" and are therefore called "Physical systems".

The word "control" means to regulate, to direct, or to command. The word "system" means a combination of devices and components connected together to perform a certain function. This system may be physical, biological, economic etc.,

"control system" is defined as combination of devices and components connected or related so as to command, direct or regulate it self or another system. It is used in many applications eg. control of temperature, liquid level, position, Velocity, flow, pressure, acceleration etc.;

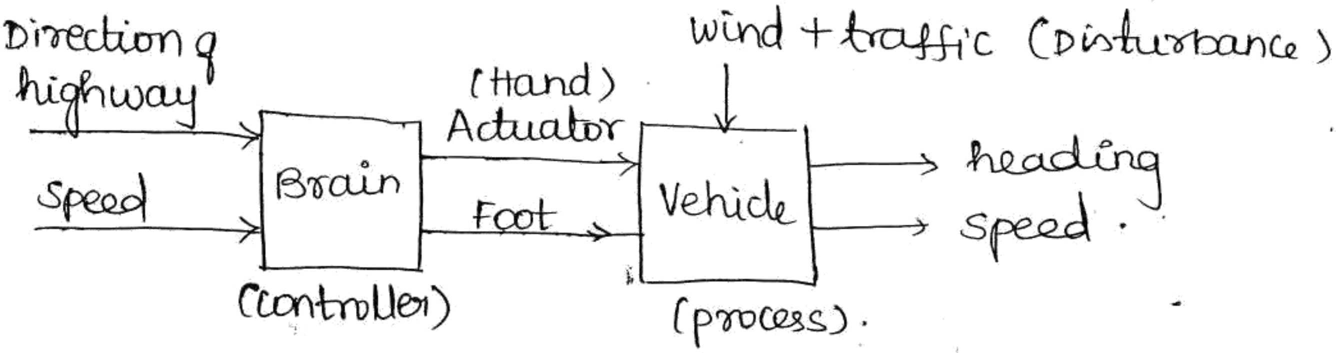
Configuration of a control system



Control system. (open loop)

eg. Automobile Driving system.

It has two inputs and two controlled outputs. Command inputs are direction of highway and speed limit with traffic signals. It can be represented as,



Control system Terminology :-

(i) Reference Input : It provides input signal for desired output

(ii) Error Detector : It is an element in which one system variable is subtracted from another variable to obtain third variable. also called comparator.

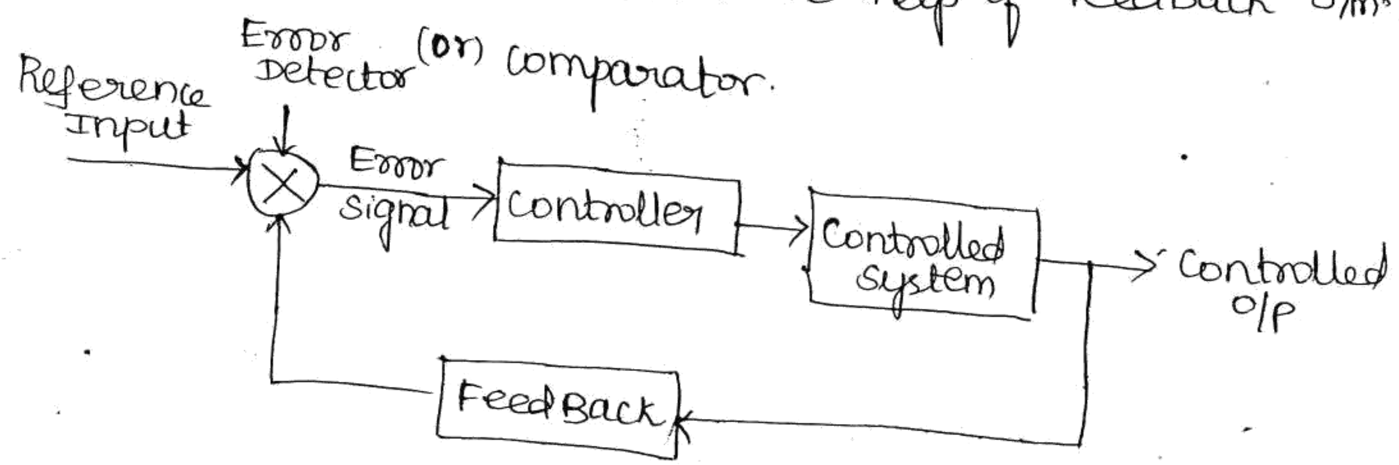
(iii) Feedback element : It measures the controlled output, convert or transforms to a suitable value for comparison with ref. Input.

(iv) Error signal: It is an algebraic sum of reference input and feedback.

(v) controller: controller is an element that is required to generate the appropriate control signal. The controller operates until the error between controlled output and desired output is reduced to zero.

(vi) controlled system:- It is a body, a plant, a process or a machine of which a particular condition is to be controlled. for eg., room heating system, spacecraft, reactor boiler.

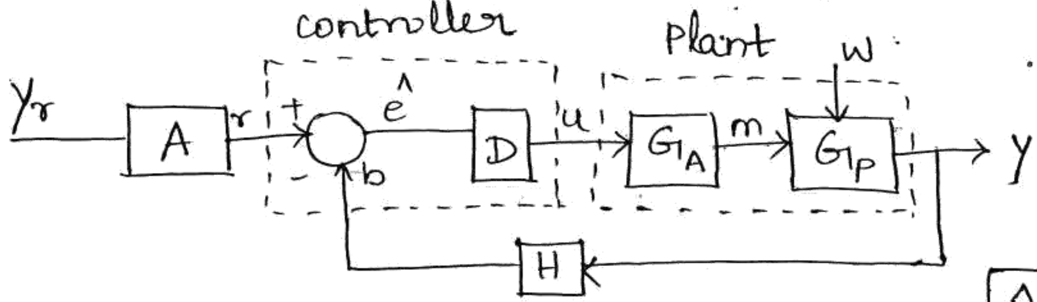
(vii) controlled output: It is produced by actuating signal available as input to the controller. controlled output is made equal to desired output with the help of feedback S/m.



"Representation of control system" (closed loop)

——— x ——— x ———

Basic structure of control system:



$$\boxed{\hat{e} = r - b}$$

Elements :-

A → Reference input element
D → control logic elements
G_A → Actuator elements
G_P → Controlled system elements
H → feedback elements
b → feedback signal.

y → controlled output
w → Disturbance input
$\hat{e}$ → Actuating error
u → control signal.
m → manipulated Variable
r → reference input
Y_r → command input.

—— × —— × ——

Classification of control systems

(i) Open and closed loop control system.
(ii) linear and non linear Control system
(iii) Time invariant and Variant control system
(iv) continous time & Discrete control system
(v) SISO & MIMO control system
(vi) lumped & Distributed parameter control system
(vii) Deterministic & Stochastic control system.
(viii) static & Dynamic control system.

(i) open loop and closed loop systems.

open loop	closed loop.
No feedback used.	Feedback is used for compare the desired output and reference input
open loop system is generally stable.	closed loop system become unstable under certain conditions
simple to develop and cheap. Accuracy is determined by calibration of their elements	It is more complex. complicated to construct and costly.
Affected by non linearities.	Adjust to effect of non linearity present in the s/m.
eg., washing machine, fixed time traffic, room heater	eg. refrigerator, servo motor, smart AC, generator o/p.

(ii) linear and non linear control system:-

If a system obeys the principle of superposition. Such a system is called linear system. The superposition principle states that response produced by simultaneous application of two different forcing function is equal to sum of individual responses.

If a system do not obey the superposition principle is called non linear system.

(iii) Time invariant and Time Varying control system

If system parameters do not vary with time. If it is independent of time at which input is

applied. such a system is called Time invariant.
eg. R, L, C.

If a system parameters vary with time. The response depend on time at which input is applied such a system is called Time Variant system.

eg., Space Vehicle control system. where mass decreases with time.

(iv) Continous time & Discrete control system:

If all system parameters are function of continous time 't'. is called continous time control System: eg., speed control of DC Motor.

If the system control involves one or more Variables that are known only at discrete instants of time is called Discrete time control system.

eg. A/D converters, generator excitation control system.

(v) SISO & MIMO:

A system with one command input and one controlled output is called Single Input single output [SISO].

A system with multiple inputs & outputs is called Multi Input - Multi output (MIMO).

eg. boiler drum level, robot arm control.

(vi) Lumped and Distributed parameter control system:-

If control systems described by ordinary Differential equations are lumped parameter. If control system described by partial Differential equations are called distributed parameter. eg. R, L, C.

(vii) Deterministic and stochastic Rontrol system:-

If the response is predictable and repeatable is called Deterministic. If response involve random Variable parameters. Such a system is called Stochastic control system.

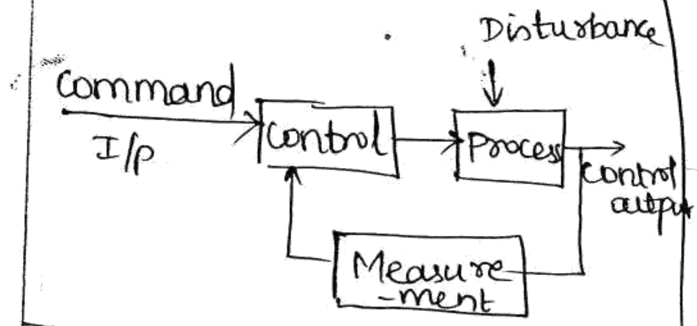
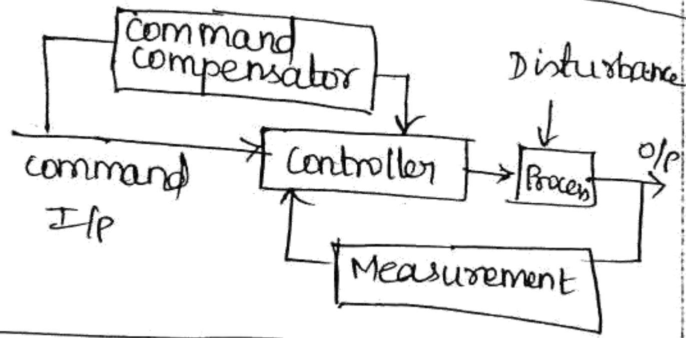
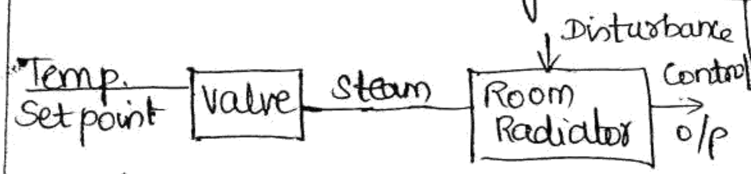
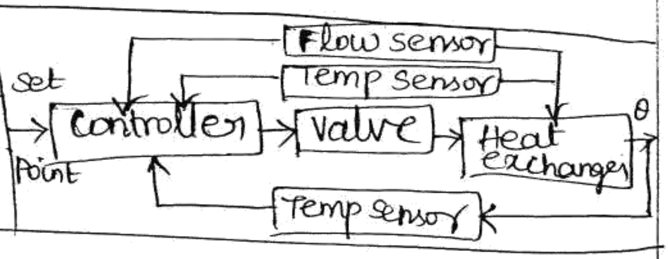
(viii) Static and Dynamic control system:-

If present output depends on past input is called dynamic or time dependent system. If present output depends only on present input is called static or time independent system.

—— × —— × ——

Feed Back & Feed Forward control theory:-

Both control technique is used to compensate the disturbance. It does not generally require any Specialized control theory. basic System dynamics are Sufficient in most cases.

Feed Back	Feed Forward
	
Main objective is to error self nulling.	Main objective is to control or minimize transient error
. It compensates the any disturbance which affecting controlled variable.	It compensate any disturbance before they affect the controlled variable
It does not include Feed Forward structure.	It includes Feedback structure
once disturbance enters a process, it must propage through process and force the controlled variable to deviate before corrective actions are taken	Disturbance are measured before they enter the process and required value of manipulated variable to maintain desired controlled variable.
eg. Residential Heating system	eg. Heat Exchanger.
	

Electrical & Mechanical Transfer function Models:

In control theory, transfer functions are commonly used to characterize the input-output relationship of components or systems that can be described by linear time-invariant differential equations.

Transfer function of a linear Time invariant system is defined as the ratio of Laplace Transform of the output to the Laplace transform of the input, under the assumptions that all initial conditions are zero.

$$\boxed{G(S) = \frac{C(S)}{R(S)} \Bigg|_{\text{zero initial conditions}}}$$

Let us consider a linear, Time invariant system defined by following differential equation:

$$[a_0 y^n + a_1 y^{n-1} + a_2 y^{n-2} + \cdots + a_{n-1} y + a_n] =$$
$$[b_0 x^m + b_1 x^{m-1} + \cdots + b_{m-1} x + b_m]$$

where $n \geq m$,

$y \rightarrow$ output of s/m

$x \rightarrow$ input of s/m.

Taking Laplace Transform, on both sides,

$$(a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n) Y(s) =$$
$$(b_0 s^m + b_1 s^{m-1} + \cdots b_{m-1} s + b_m) X(s).$$

Transfer function: $G(s) = \left. \dfrac{L(\text{output})}{L(\text{input})} \right|_{\substack{\text{initial} \\ \text{condition} = 0}}$.

$$\boxed{G(s) = \frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}}.$$

using the concept of Transfer function,

(i) calculate $\boxed{\text{order}}$ of system by knowing highest power of s in denominator.

(ii) calculate $\boxed{\text{Type}}$ of system by number of open-loop poles at the origin.

Always order of system is $\geq$ its type

$\boxed{\text{order} \geq \text{type}}$.

— × — × —

1. The Transfer function of a system is given by

$$G(s) = \frac{K(s+6)}{s(s+2)(s+5)(s^2+7s+12)}$$. Determine

a) zeros, (b) poles, (c) characteristic eqn. (iv) pole-zero diagram.

Solution :-

(i) zeros:

Numerator terms equal to zero.

$\therefore \; s+6=0$

$$\boxed{s=-6}$$

(ii) poles:

Denominator terms equal to zero.

$$s(s+2)(s+5)(s^2+7s+12)=0$$

$$s(s+2)(s+5)(s+4)(s+3)=0$$

$$\therefore \; \boxed{s=0,-2,-5,-4,-3}$$

(iii) characteristic equation:

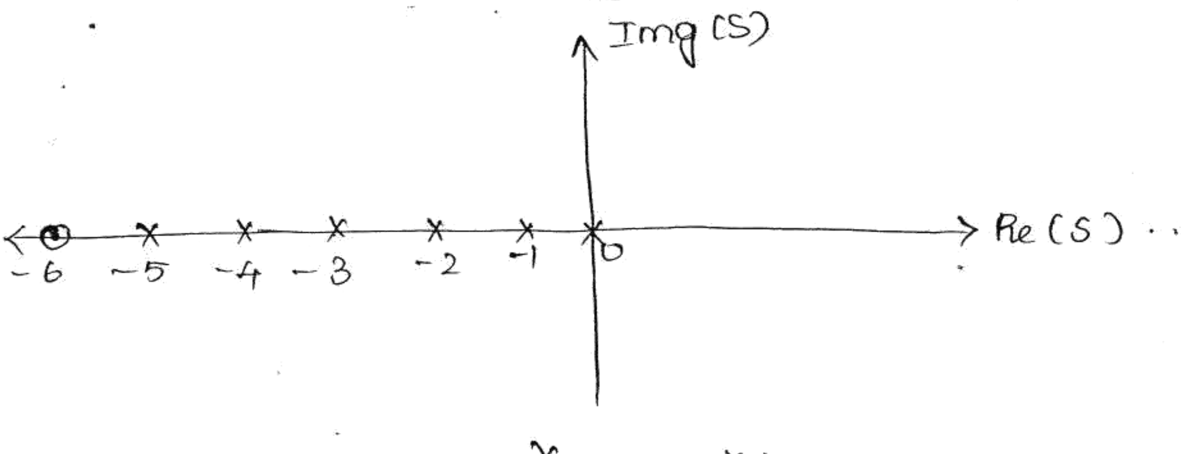
$$s(s+2)(s+5)(s^2+7s+12)=0$$

$$\Rightarrow (s^2+2s)(s^3+7s^2+12s+5s^2+35s+60)=0.$$

$$\Rightarrow s^5+5s^4+7s^4+35s^3+12s^3+60s^2+2s^4+10s^3+14s^3+70s^2+24s^2+120s=0.$$

$$\Rightarrow \boxed{s^5+14s^4+71s^3+154s^2+120s=0}$$

(iv) pole-zero Diagram:



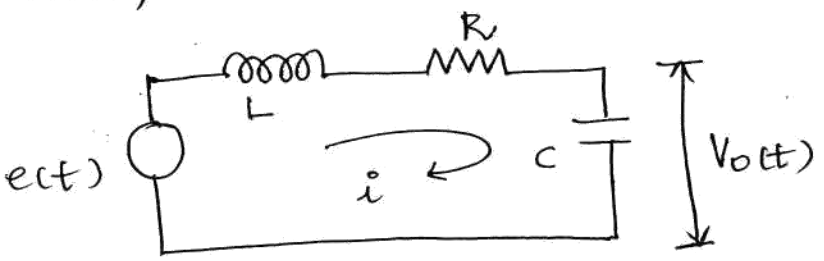
— X — X —

MODELLING OF ELECTRICAL SYSTEM:

Resistor, Inductor, capacitor are the three basic elements of an electric circuit. The circuit is analysed by application of Kirchhoff Voltage and current laws.

Sno	Element	Symbol	Time domain V(t)=	Laplace Domain V(S)=
1.	Resistor	i(t) → R, + V(t) −	$i(t) \times R$	$R\,I(s)$
2.	Inductor	i(t) → L, + V(t) −	$L \cdot \frac{d\,i(t)}{dt}$	$LS\,I(s)$
3.	capacitor	i(t) → C, + V(t) −	$\frac{1}{C}\int i(t)\,dt$	$\frac{1}{SC}\,I(s)$

1. Consider an electrical system of RLC series circuit, Find out Transfer function:



⇒ solution:

system equation:

$$e(t) = L \cdot \frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt \longrightarrow (1) \text{ (Apply KVL)}$$

Taking Laplace Transform on both sides.

$$E(S) = LSI(S) + RI(S) + \frac{1}{CS}\,I(s).$$

$$E(S) = \left[LS + R + \frac{1}{CS}\right] I(S) \longrightarrow (2)$$

Assuming all initial conditions to be zero.

$$E(s) = \left[Ls + R + \frac{1}{Cs}\right] I(s)$$

$$\boxed{E(s) = \left[\frac{LCs^2 + RCs + 1}{Cs}\right] I(s)} \longrightarrow (3)$$

Let the output Voltage $V_o(t)$ be taken across the capacitor, C. Then,

$$V_o(t) = \frac{1}{C}\int i\,dt \longrightarrow (4)$$

Taking Laplace Transform on both sides,

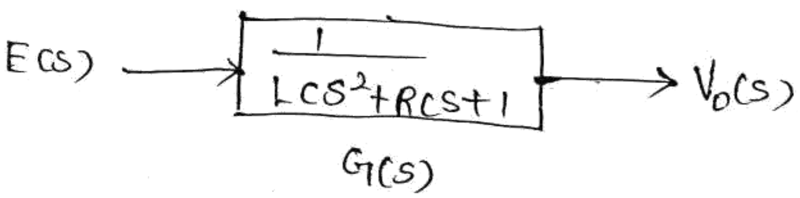
$$\boxed{V_o(s) = \frac{1}{Cs} I(s)} \longrightarrow (5)$$

∴ Transfer function is given by,

$$G(s) = \frac{V_o(s)}{E(s)} = \frac{\cancel{I(s)}/Cs}{\left[(LCs^2 + RCs + 1)/Cs\right]\cancel{I(s)}}$$

$$\boxed{G(s) = \frac{V_o(s)}{E(s)} = \frac{1}{LCs^2 + RCs + 1}}$$

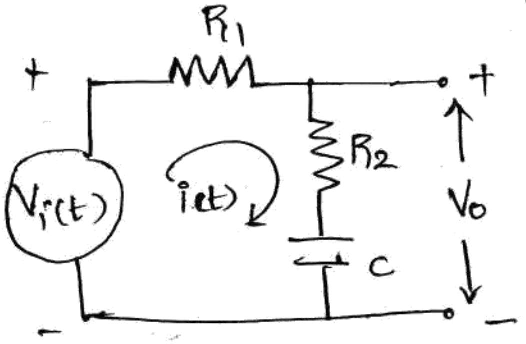
Block Diagram Representation:-



This is second order system since highest power of s in the denominator is 2.

— x — x —

2. Determine the transfer function of electrical system as shown in figure.



= Solution:

Loop equation :- (Apply KVL)

$$V_i(t) = R_1 i(t) + R_2 i(t) + \frac{1}{C}\int i(t)dt \longrightarrow (1)$$

Taking Laplace Transform on both sides.

$$V_i(S) = R_1 I(S) + R_2 I(S) + \frac{1}{CS} I(S).$$

$$\boxed{V_i(S) = \left[R_1 + R_2 + \frac{1}{CS}\right] I(S)} \longrightarrow (2)$$

Output equation:-

$$V_o(t) = R_2 i(t) + \frac{1}{C}\int i(t)\, dt \longrightarrow (3)$$

Taking Laplace Transform on both sides.

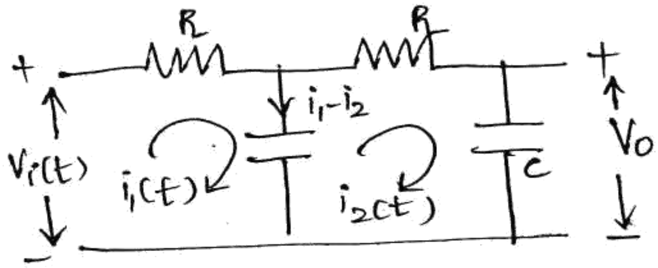
$$V_o(S) = R_2 I(S) + \frac{1}{CS} I(S).$$

$$\boxed{V_o(S) = \left[R_2 + \frac{1}{CS}\right] I(S)} \longrightarrow (3)$$

Transfer function: $\dfrac{V_o(S)}{V_i(S)} = \dfrac{\left[R_2 + \frac{1}{CS}\right] \cancel{I(S)}}{\left[R_1 + R_2 + \frac{1}{CS}\right] \cancel{I(S)}}$

$$\boxed{T.F = \frac{V_o(s)}{V_i(s)} = \frac{R_2Cs+1}{R_1Cs+R_2Cs+1}}$$

3. Obtain the transfer function for electrical network.



= Solution:

Apply KVL to loop 1:

$$V_i(t) = Ri_1(t) + \frac{1}{C}\int[i_1(t) - i_2(t)]\,dt \longrightarrow (1)$$

Taking Laplace transform on both sides.

$$V_i(s) = RI_1(s) + \frac{1}{Cs}[I_1(s) - I_2(s)]$$

$$V_i(s) + \frac{1}{Cs}I_2(s) = \left[R + \frac{1}{Cs}\right]I_1(s)$$

$$I_1(s) = \frac{[V_i(s)\,Cs + I_2(s)]/\cancel{Cs}}{[1+RCs]/\cancel{Cs}}$$

$$\boxed{I_1(s) = \frac{V_i(s)\,Cs + I_2(s)}{1+RCs}} \longrightarrow (2)$$

Apply KVL to loop 2,

$$R\,i_2(t) + \frac{1}{C}\int i_2(t)\,dt + \frac{1}{C}\left[\int i_2(t) - i_1(t)\right]dt = 0 \longrightarrow (3)$$

Taking Laplace Transform on both sides,

$$R I_2(s) + \frac{1}{Cs} I_2(s) + \frac{1}{Cs}[I_2(s) - I_1(s)] = 0$$

$$I_2(s)\left[R + \frac{1}{Cs} + \frac{1}{Cs}\right] - \frac{1}{Cs} I_1(s) = 0$$

$$I_2(s)\left(\frac{2+RCs}{Cs}\right) = \frac{1}{Cs} I_1(s) \longrightarrow (4)$$

Substitute $\boxed{I_1(s) = \frac{V_i(s)\,Cs + I_2(s)}{1+RCs}}$

$\therefore$ Eq (4) becomes,

$$I_2(s)\left(\frac{2+RCs}{Cs}\right) = \frac{1}{Cs}\left[\frac{V_i(s)\,Cs + I_2(s)}{1+RCs}\right]$$

$$I_2(s)\left(\frac{2+RCs}{Cs}\right) = \frac{V_i(s)}{1+RCs} + \frac{I_2(s)}{Cs(1+RCs)}$$

$$I_2(s)\left[\frac{2+RCs}{Cs} - \frac{1}{Cs(1+RCs)}\right] = \frac{V_i(s)}{1+RCs}$$

$$I_2(s)\left[\frac{(2+RCs)(1+RCs)-1}{Cs\,\cancel{(1+RCs)}}\right] = \frac{V_i(s)}{\cancel{1+RCs}}$$

$$\boxed{I_2(s) = \frac{V_i(s)\,Cs}{1+3RCs+R^2C^2s^2}}$$

$$V_i(s) = \frac{I_2(s)}{Cs}\left[1+3RCs+R^2C^2s^2\right] \longrightarrow (5)$$

Output equation:- $V_o(t) = \frac{1}{C}\int i_2(t)\,dt \rightarrow (6)$

Taking Laplace Transform,

$$\boxed{V_o(s) = \frac{1}{CS} I_2(s)} \rightarrow (7).$$

Transfer function: $G(s) = \frac{V_o(s)}{V_i(s)} = \frac{\cancel{I_2(s)}/\cancel{CS}}{\cancel{I_2(s)}(1+3RCS+R^2C^2S^2)/\cancel{CS}}$

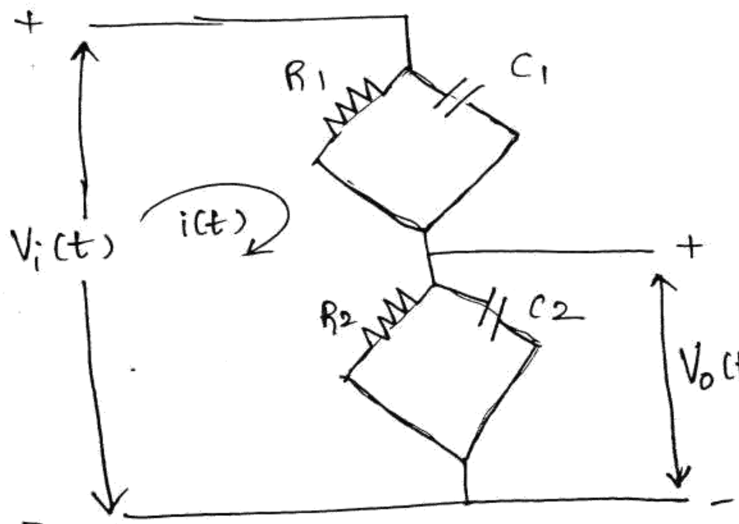
$$\therefore \boxed{G(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{1+3RCS+R^2C^2S^2}}$$

Block Diagram Representation:-

$$V_i(s) \longrightarrow \boxed{G(s) = \frac{1}{1+3RCS+R^2C^2S^2}} \longrightarrow V_o(s).$$

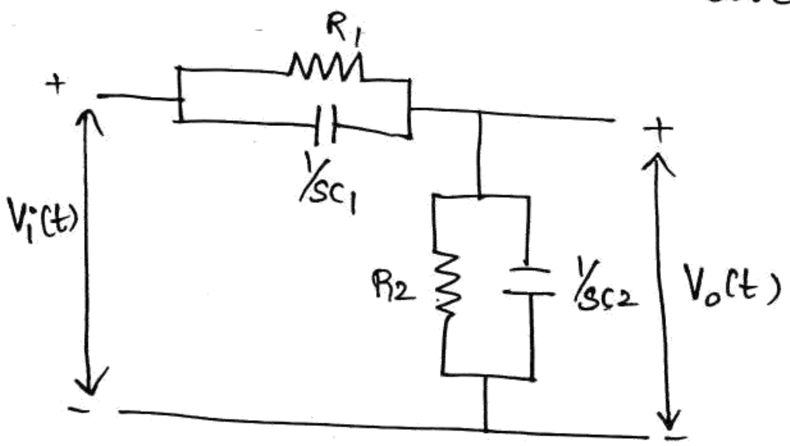
—— x —— x ——

4. Obtain Transfer function for following electrical Network.



Solution:-

Step 1 :- Redrawn the circuit in terms of 's' domain.



step 2 :-

Apply Voltage divider rule,

$$V_o(s) = V_i(s)\,\frac{\left(R_2 \,\|\, \frac{1}{sC_2}\right)}{\left(R_1 \,\|\, \frac{1}{sC_1}\right) + \left(R_2 \,\|\, \frac{1}{sC_2}\right)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\dfrac{R_2/sC_2}{R_2 + \frac{1}{sC_2}}}{\left(\dfrac{R_1/sC_1}{R_1 + \frac{1}{sC_1}}\right) + \left(\dfrac{R_2/sC_2}{R_2 + 1/sC_2}\right)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2/R_2C_2s + 1}{\dfrac{R_1}{1+R_1C_1s} + \dfrac{R_2}{1+R_2C_2s}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2/1 + \cancel{R_2}C_2S}{\dfrac{R_1(1+R_2C_2S) + R_2(1+R_1C_1S)}{(1+R_1C_1S)(1+\cancel{R_2C_2S})}} = \frac{R_2(1+R_1C_1S)}{R_1(1+R_2C_2S) + R_2(1+R_1C_1S)}.$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2(R_1C_1S+1)}{R_1(1+R_2C_2S)+R_2(1+R_1C_1S)}$$

$\therefore$ Transfer function $G(S) = \dfrac{V_o(S)}{V_i(S)} = \dfrac{R_2 + R_1R_2C_1S}{R_1 + R_2 + SR_1R_2C_2 + SR_1R_2C_1}$ //.

$$V_i(s) \longrightarrow \boxed{\dfrac{R_2 + R_1R_2C_1S}{R_1 + R_2 + SR_1R_2C_2 + SR_1R_2C_1}} \longrightarrow V_o(s).$$

Block Diagram Representation.

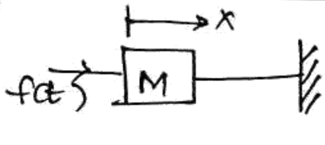
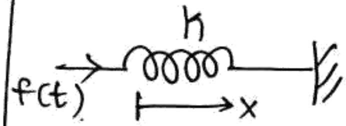
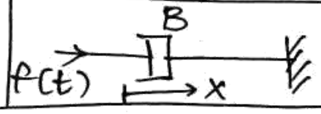
—— × —— × ——

Modelling of Mechanical systems.

The equations of motions are generally formulated using Newtons law of motion. A Mechanical system may have either

Translational Motion	Rotational Motion
[Motion take place in straight lines]	[Motion will be in rotation]
I/p → Force (Newton)	I/P → Torque (N-m)
o/p → Displacement. (metre)	o/p → Angular Displacement (θ).
1. Mass (M) (kg)	1. Moment of Inertia (J)
2. Spring (k) (N/m)	2. Shaft Stiffness (K)
3. Damper (B) (N/rad/sec)	3. Viscous friction (B) coefficient.

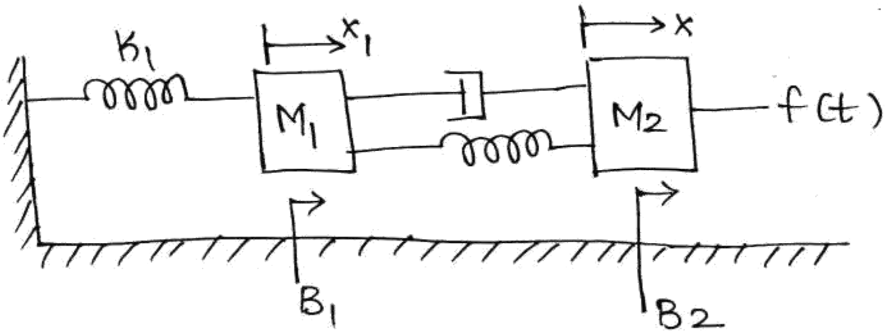
Translational Mechanical system.

Sno	Element	symbol	Representation	Time domain	Laplace Domain
1	Mass	M		$f(t) = M\frac{d^2x}{dt^2}$	$F(S) = MS^2X(S)$
2	Spring	K		$f(t) = Kx$	$F(S) = K X(S)$
3	Damper	B		$f(t) = B.\frac{dx}{dt}$	$F(S) = BSX(S)$

<u>Mass</u> stores Translational kinetic Energy due to its Velocity. while <u>spring</u> stores Translational potential Energy which is due to its position. <u>Damper</u> or Dashpot is capable of dissipating energy.

—— × —— × ——

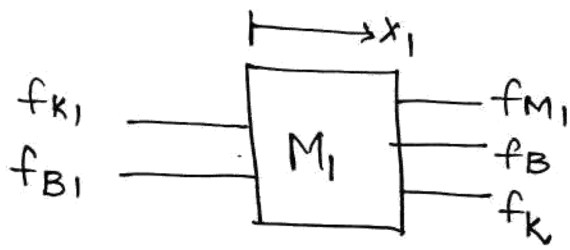
1. write the Differential Equation governing the translational Mechanical system as shown in Fig. & determine the Transfer function.



<u>Hint</u> :- $\boxed{T.F = \frac{X(S)}{F(S)}}$

Solution:-

Step 1: Draw the free body diagram of Mass element 'M_1'.



By using Newtons II law of Motion,

Sum of opposing force = sum of Applying Force.

$$f_{M_1} + f_B + f_K + f_{K_1} + f_{B_1} = 0.$$

Differential equation:-

$$\boxed{M_1 \frac{d^2x_1}{dt^2} + B\frac{d}{dt}(x_1 - x) + K(x_1 - x) + K_1x_1 + B_1\frac{dx_1}{dt} = 0}$$

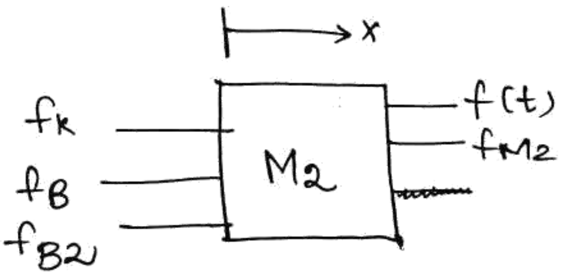
Step 2:- Taking Laplace Transform on both sides.

$$M_1 s^2 X_1(s) + Bs[X_1(s) - X(s)] + K[X_1(s) - X(s)] + K_1 X_1(s) + B_1 s X_1(s) = 0.$$

$$X_1(s)[M_1s^2 + K_1 + B_1s + Bs + K] - X(s)[Bs + K] = 0.$$

$$\boxed{X_1(s) = \frac{X(s)(Bs + K)}{M_1s^2 + (K_1 + K) + (B_1 + B)s}}$$

Step 3:- Draw the free Body diagram of Mass element 'M$_2$'.



By, Netwons II law of motion:-

$$f(t) = f_{M_2} + f_K + f_B + f_{B2}$$

$$\boxed{f(t) = M_2 \frac{d^2x}{dt^2} + K(x - x_1) + B\frac{d}{dt}(x - x_1) + B_2\frac{dx}{dt}}$$

Step 4: Taking Laplace Transform on both sides,

$$F(s) = M_2 s^2 X(s) + K(X(s) - X_1(s)) + BS(X(s) - X_1(s)) + B_2 S X(s).$$

$$F(s) = [M_2 s^2 + (B_2 S + BS) + K]X(s) - [BS + K]X_1(s).$$

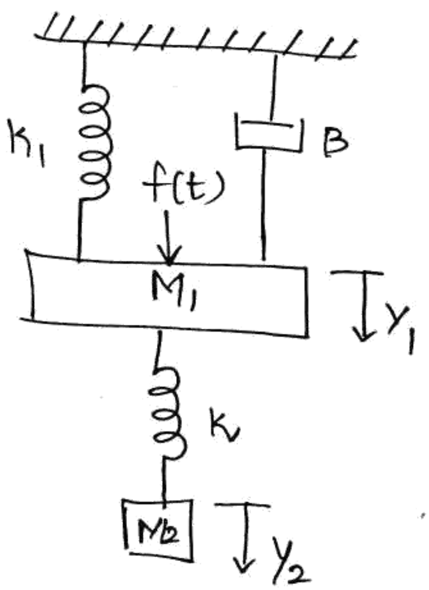
Substitute $X_1(s)$ Value to above equation.

$$F(s) = [M_2 s^2 + (B_2 S + BS) + K]X(s) - [BS + K]\left[\frac{X(s)(BS+K)}{M_1 s^2 + (K_1 + K) + (B_1 + B)S}\right]$$

$$F(s) = \left[(M_2 s^2 + (B_2 S + BS) + K) - \frac{(BS+K)^2}{M_1 s^2 (K_1 + K)(B_1 + B)S}\right] X(s)$$

$$\therefore T.F = G(s) = \frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B_1 + B)S + (K_1 + K)}{[M_2 s^2 + (B_2 S + BS) + K)(M_1 s^2 (K_1 + K)(B_1 + B)S - (BS + K)^2]}$$ //

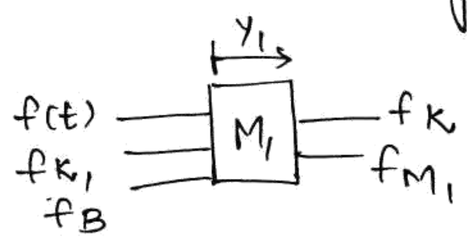
2. Determine Transfer function for given diagram.



(∵ Hint : $T.F = \dfrac{Y_2(s)}{F(s)}$)

⇒ Solution :-

<u>Step 1</u> :- Draw Free body Diagram of mass 'M_1' :-



By Newtons II law,

$$f(t) = f_{K_1} + f_B + f_K + f_{M_1}$$

$$\boxed{f(t) = K_1 Y_1 + B\frac{dY_1}{dt} + K(Y_1 - Y_2) + M_1\frac{d^2Y_1}{dt^2}}$$

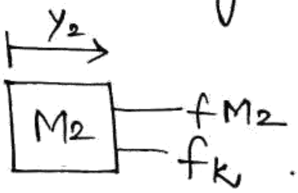
<u>Step 2</u> :- Apply Laplace Transform on both sides,

$$F(s) = K_1 Y_1(s) + BsY_1(s) + K[Y_1(s) - Y_2(s)] + M_1 s^2 Y_1(s)$$

$$F(s) = [K_1 + Bs + K + M_1 s^2]\, Y_1(s) - K Y_2(s)$$

$$Y_1(s) = \underline{F(s) + KY_2(s)} \longrightarrow (1)$$

Step 3 :- Draw Free body Diagram for mass 'M_2':



By Newtons II law of Motion,

$$f_{M_2} + f_k = 0$$

$$M_2 \cdot \frac{d^2 y_2}{dt^2} + K(y_2 - y_1) = 0.$$

Step 4 :- Apply Laplace transform,

$$M_2 s^2 Y_2(s) + K(Y_2(s) - Y_1(s)) = 0.$$

$$[M_2 s^2 \cancel{+} K] Y_2(s) - K Y_1(s) = 0.$$

$$(M_2 s^2 + K) Y_2(s) - K\left(\frac{F(s) + K Y_2(s)}{M_1 s^2 + Bs + (K_1 + K)}\right) = 0.$$

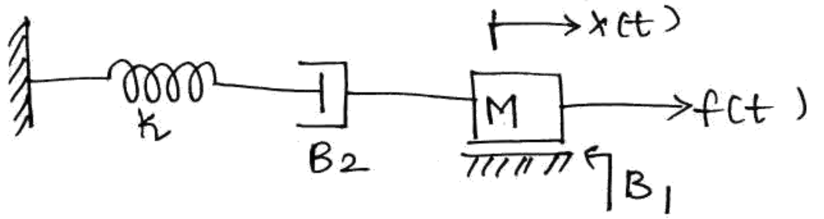
$$Y_2(s)\left[M_2 s^2 + K - \frac{K^2}{M_1 s^2 + Bs + K_1 + K}\right] = \frac{K F(s)}{M_1 s^2 + Bs + K_1 + K}$$

Transfer function:- (Take L.C.M & Denom. get cancel)

$$\boxed{G(s) = \frac{Y_2(s)}{F(s)} = \frac{K}{(M_2 s^2 + K)(M_1 s^2 + Bs + K_1 + K) - K^2}}$$

—— X —— X ——

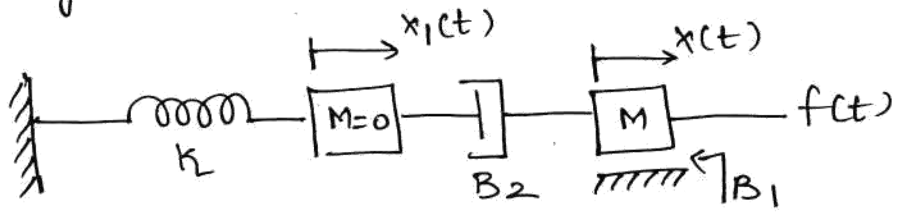
3. Determine Transfer function for the system shown in Figure,



= Solution:-

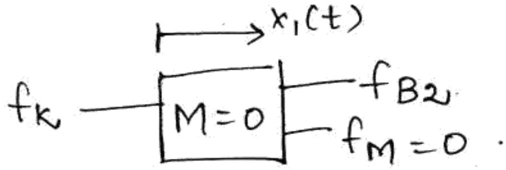
let x_1 be the displacement at meeting point of spring and dashpot. Hence system has two nodes, mass M & meeting point of spring and dashpot.

∴ Diagram becomes,



Step 1 :-

Free Body diagram of mass $\boxed{M=0}$.



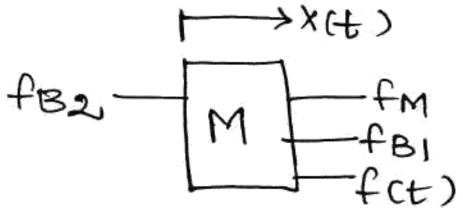
$$0 = f_K + f_{B_2}$$

$$0 = K x_1(t) + B_2 \frac{d}{dt}(x_1(t) - x(t))$$

Taking L.T $\quad (B_2 s + K) X_1(s) - B_2 s X(s) = 0$.

$$\boxed{X_1(s) = \left[\frac{B_2 s}{B_2 s + K}\right] X(s)}.$$

step 3:- Draw Free body diagram of mass element M,



By Newtons II law of motion:-

$$f(t) = f_M + f_{B1} + f_{B2}.$$

$$f(t) = M\frac{d^2x}{dt^2} + B_1\frac{dx}{dt} + B_2\frac{d(x-x_1)}{dt}.$$

Taking Laplace Transform on both sides,

$$Ms^2X(s) + B_1sX(s) + B_2s[X(s) - X_1(s)] = F(s).$$

$$[Ms^2 + (B_1 + B_2)s]X(s) - B_2sX_1(s) = F(s).$$

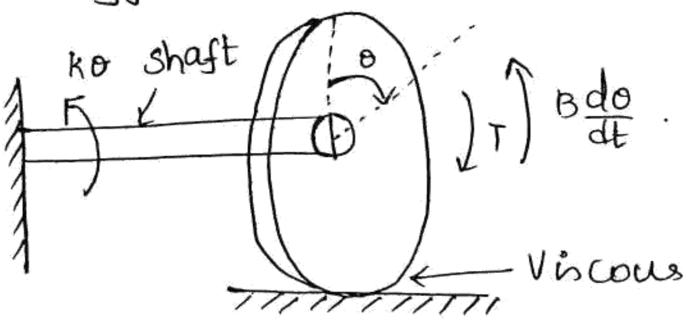
Substitute $X_1(s)$ value to above equation.

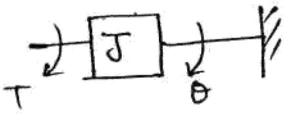
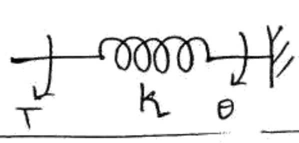
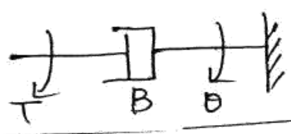
$$[Ms^2 + (B_1 + B_2)s]X(s) - B_2s\left[\frac{B_2s}{B_2s+K}\right]X(s) = F(s).$$

$$\boxed{T.F = \frac{X(s)}{F(s)} = \frac{B_2s+K}{[Ms^2 + (B_1+B_2)s](B_2s+K) - (B_2s)^2}}$$

Rotational Mechanical System.

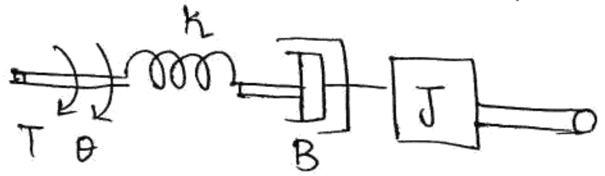
Mechanical systems involving rotation around a fixed axis are often seen in Machineries such as turbines, pumps, rotating discs, gears, generators, motor and so on. It consists of rotating disc of moment of inertia J, shaft stiffness K. The disc rotates in a Viscous medium with a viscous friction coefficient B.



Sno	Element	Symbol	Time ~~do~~ Representation	Time domain	Laplace domain
1	Moment of inertia	J		$T = J\frac{d^2\theta}{dt^2}$	$T(S) = JS^2\theta(S)$
2	shaft stiffness	K		$T = K\theta$	$T(\theta) = K\theta(S)$
3	Viscous friction coefficient	B		$T = B\frac{d\theta}{dt}$	$T(S) = BS\theta(S)$

$$\boxed{\text{Applied Torque} = \text{Inertia Torque} + \text{Damping Torque} + \text{Torsional Torque}}$$

1.



Determine Transfer function for given Rotational mechanical system.

Solution :-

$$T = T_K + T_B + T_J$$

$$T = K\theta + B\frac{d\theta}{dt} + J\frac{d^2\theta}{dt^2} \longrightarrow (1)$$

Taking Laplace Transform on both sides,

$$T(s) = K\theta(s) + BS\,\theta(s) + Js^2\theta(s)$$

$$T(s) = [K + BS + Js^2]\,\theta(s) \longrightarrow (2)$$

Transfer function : $G(s) = \frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + BS + K}$

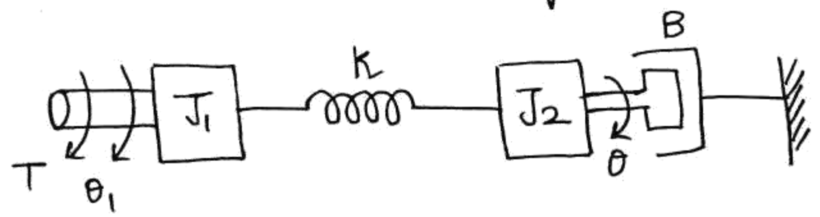
$$\boxed{G(s) = \frac{1}{Js^2 + BS + K}}$$

Block Diagram Representation :-

$$T(s) \longrightarrow \boxed{G(s) = \frac{1}{Js^2 + BS + K}} \xrightarrow{\theta(s)}$$

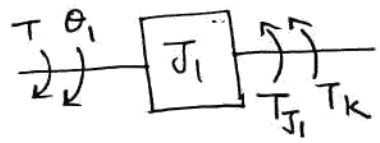
Rotational Mechanical system.

1. Determine Transfer function for the following Mechanical Rotational system.



Solution:

Step 1: Draw Free Body diagram of moment of inertia element 'J_1'.



By using Newtons II law of motion.

Sum of opposing torque = sum of Applying torque

$$T_{J_1} + T_K = T$$

$$\boxed{J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) = T} \longrightarrow (1)$$

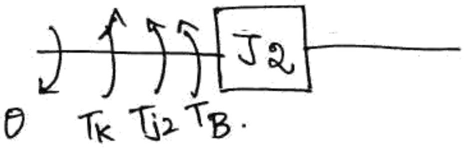
Step 2:- Taking Laplace Transform on bothsides,

$$J_1 s^2 \theta_1(s) + K[\theta_1(s) - \theta(s)] = T(s).$$

$$\theta_1(s)[J_1 s^2 + K] - K\theta(s) = T(s).$$

$$\boxed{\theta_1(s) = \frac{T(s) + K\theta(s)}{J_1 s^2 + K}} \longrightarrow (2)$$

Step 3: Draw free body diagram of moment of inertia element J_2.



By using Newtons II law of motion,

$$T_{j2} + T_B + T_K = 0.$$

$$\boxed{J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0.} \longrightarrow (3)$$

Step (4): Taking Laplace Transform on both sides.

$$J_2 s^2 \theta[s] + Bs\,\theta(s) + K(\theta(s) - \theta_1(s)) = 0.$$

$$\boxed{\theta(s)[J_2 s^2 + Bs + K] - K\theta_1(s) = 0.} \longrightarrow (4)$$

Substitute $\theta_1(s)$ Value in Eq (4).

$$\theta(s)[J_2 s^2 + Bs + K] - K\left[\frac{T(s) + K\theta(s)}{J_1 s^2 + K}\right] = 0.$$

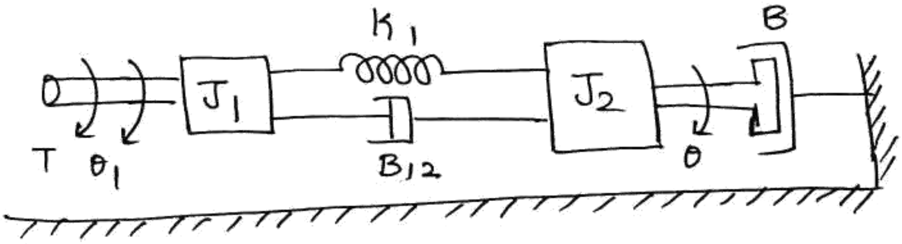
$$\theta(s)[J_2 s^2 + Bs + K] - \frac{KT(s)}{J_1 s^2 + K} - \frac{K^2\theta(s)}{J_1 s^2 + K} = 0$$

$$\theta(s)\left[J_2 s^2 + Bs + K - \frac{K^2}{J_1 s^2 + K}\right] = \frac{K\,T(s)}{J_1 s^2 + K}.$$

Step 5:- Transfer function : $\boxed{G(s) = \frac{\theta(s)}{T(s)}}$

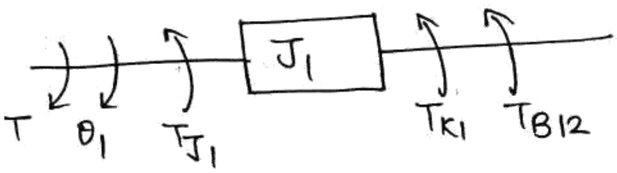
$$\boxed{G(s) = \frac{\theta(s)}{T(s)} = \frac{K}{[J_2 s^2 + Bs + K][J_1 s^2 + K] - K^2}}$$ //.

2. write the Differential equation for the system shown in Figure & determine Transfer function.



= step 1 :-

Draw Free Body diagram of moment of inertia J1



By using Newtons II law of motion,

$$T_{J_1} + T_{K_1} + T_{B_{12}} = T$$

$$J_1 \frac{d^2\theta_1}{dt^2} + K_1(\theta_1 - \theta) + B_{12}\frac{d}{dt}(\theta_1 - \theta) = T \longrightarrow (1)$$

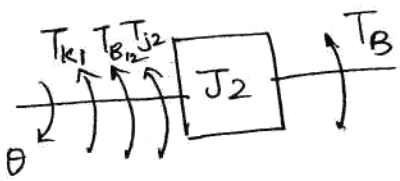
step 2 :- Taking Laplace Transform on both sides,

$$J_1 s^2 \theta_1(s) + K_1[\theta_1(s) - \theta(s)] + B_{12}[\theta_1(s) - \theta(s)] = T(s).$$

$$\theta_1(s)[J_1 s^2 + K_1 + B_{12} s] - \theta(s)[K_1 + B_{12} s] = T(s).$$

$$\boxed{\theta_1(s) = \frac{T(s) + \theta(s)[K_1 + B_{12} s]}{J_1 s^2 + B_{12} s + K_1}} \longrightarrow (2)$$

Step 3 :- Draw free Body diagram of moment of inertia 'J_2'.



By using Newtons II law of motion,

$$T_{J2} + T_B + T_{K1} + T_{B12} = 0.$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K_1(\theta - \theta_1) + B_{12}\frac{d}{dt}(\theta - \theta_1) = 0 \longrightarrow (3)$$

Step 4 :- Taking Laplace Transform on both sides,

$$J_2 s^2 \theta(s) + Bs\theta(s) + K_1[\theta(s) - \theta_1(s)] + B_{12}s[\theta(s) - \theta_1(s)] = 0$$

$$\theta(s)[J_2 s^2 + Bs + K_1 + B_{12}s] - \theta_1(s)[K_1 + B_{12}s] = 0 \longrightarrow (4)$$

Substitute $\theta_1(s)$ from eq (2) in eq (4).

$$\theta(s)[J_2 s^2 + Bs + K_1 + B_{12}s] - \left[\frac{T(s) + \theta(s)[K_1 + B_{12}s]}{J_1 s^2 + B_{12}s + K_1}\right](K_1 + B_{12}s) = 0$$

$$\theta(s)\left[\frac{[J_1 s^2 + B_{12}s + k][J_2 s^2 + Bs + B_{12}s + K_1] - [K_1 + B_{12}s]^2}{[J_1 s^2 + \cancel{B_{12}s} + K_1]}\right] = \frac{T(s)[K_1 + B_{12}s]}{J_1 s^2 + \cancel{B_{12}}s + K_1}$$

Transfer function :-
$$\boxed{\frac{\theta(s)}{T(s)} = \frac{K_1 + B_{12}s}{[J_1 s^2 + B_{12}s + k][J_2 s^2 + Bs + B_{12}s + K_1] - [K_1 + B_{12}s]^2}}$$

ANALOGIOUS SYSTEM
(similar)

Sometimes Mechanical and other systems are converted into electrical analogous systems for the easy of design, modification and analysis. Analogous systems have same type of differential equations. There are four types of Analogies namely,

(i) Force - Voltage Analogy

(ii) Force - current Analogy

(iii) Torque - Voltage Analogy

(iv) Torque - current Analogy.

The Translational Mechanical system is represented by mass, spring and damper in equation,

$$\boxed{M\frac{d^2x}{dt^2} + B\cdot\frac{dx}{dt} + Kx = f(t)} \longrightarrow (1)$$

The Rotational Mechanical system is represented by moment of inertia, ~~spring~~ shaft and viscous friction coefficient in equation,

$$\boxed{J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta = T} \longrightarrow (2)$$

The Electrical system is represented by Inductor, Resistor, capacitor in terms of voltage

$$V(t) = L\frac{di(t)}{dt} + R\,i(t) + \frac{1}{C}\int i\,dt.$$

But $\boxed{i(t) = \frac{dq}{dt}}$ ie, current is rate of flow of charge.

$$\therefore \boxed{V(t) = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q} \rightarrow (3)$$

The electrical system can be represented by capacitor, Resistor, inductor in terms of current.

$$\boxed{i(t) = \frac{1}{L}\int e\,dt + C\frac{de}{dt} + \frac{1}{R}e}$$

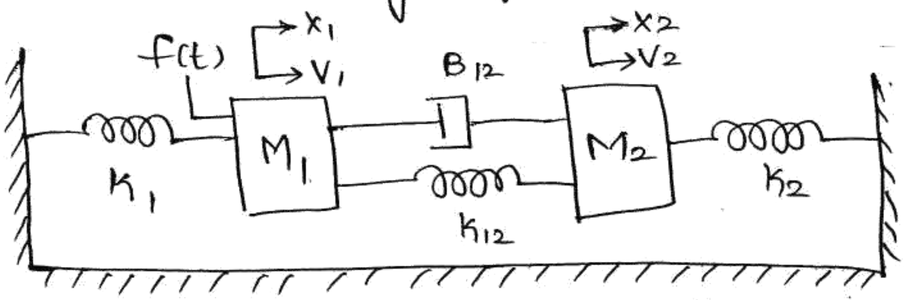
But $\boxed{e = \frac{d\psi}{dt}}$ where $\psi \rightarrow$ flux linkage.

$$\boxed{i(t) = C\frac{d^2\psi}{dt^2} + \frac{1}{R}\frac{d\psi}{dt} + \frac{1}{L}\psi} \rightarrow (4)$$

compare equation (1), (2), (3), (4) which gives equivalent parameters. It is given by Table.

Sno	Translational	Rotational	Electrical s/m in terms of Voltage	Electrical s/m in terms of I
1	F	T	$V(t)$	$i(t)$
2	M	J	L	C
3	B	B	R	$1/R$
4	K	K	$1/C$	$1/L$
5	X	θ	q	ψ
6	V	ω	$i(t)$	$V(t)$

1. Draw the Force-Voltage & Force current analogy for the following system as shown in Fig.

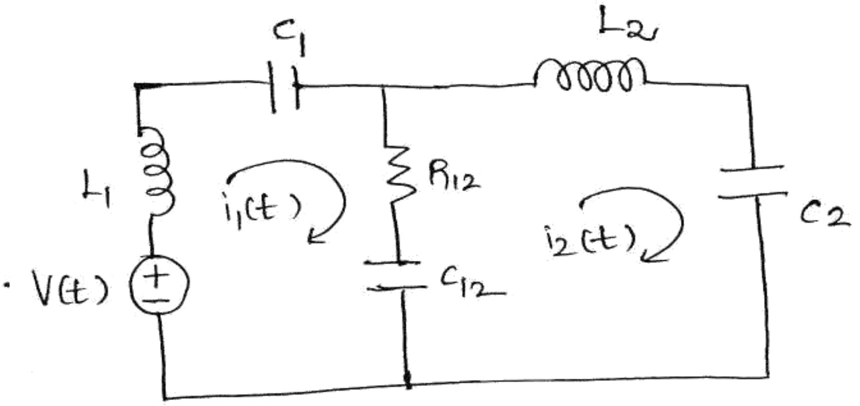


Step 1 :-

Identify equivalent electrical systems in terms of voltage.

$f(t) = V(t)$	$B_{12} = R_{12}$.	$K_2 = 1/C_2$
$K_1 = 1/C_1$	$K_{12} = 1/C_{12}$	
$M_1 = L_1$	$M_2 = L_2$	
$V_1 \rightarrow i_1(t)$	$V_2 = i_2(t)$	

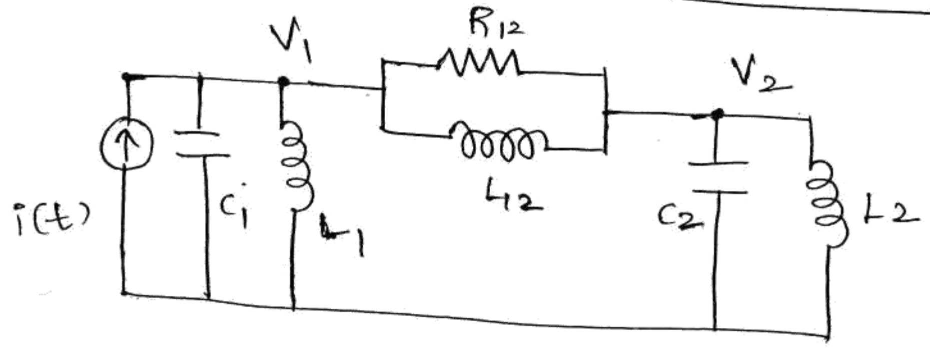
Step 2 :- Force - Voltage circuit diagram :-



Step 3: Identify equivalent electrical terms in terms of current.

$f(t) = i(t)$	$K_1 = 1/L_1$
$V_1 = V_1(t)$	$K_2 = 1/L_2$
$V_2 = V_2(t)$	$K_{12} = 1/L_{12}$
$M_1 = C_1$	$B_{12} = 1/R_{12}$
$M_2 = C_2$	

Step 4:- Force - Current circuit Diagram.



2. Write a differential equation for mechanical system as shown in Figure. Draw Force - Voltage & Force - current electrical analogy circuit.