$$C = \frac{\omega_n^2}{-\xi_\rho\omega_n - \omega_n\sqrt{\xi_\rho^2-1}\left[-2\omega_n\sqrt{\xi_\rho^2-1}\right]}$$

$$\boxed{C = \frac{1}{2\sqrt{\xi_\rho^2-1}\,\left(\xi_\rho+\sqrt{\xi_\rho^2-1}\right)}}$$

substitute A, B, C Values in eq (i)

$$C(s) = \frac{1}{s} + \frac{1}{2\sqrt{\xi_\rho^2-1}\,(-\xi_\rho+\sqrt{\xi_\rho^2-1})}\left(\frac{1}{s-s_1}\right) + \frac{1}{2\sqrt{\xi_\rho^2-1}\,(\xi_\rho+\sqrt{\xi_\rho^2-1})}\left(\frac{1}{s-s_2}\right)$$

Taking inverse laplace Transform

$$\boxed{c(t) = 1 + \frac{1}{2\sqrt{\xi_\rho^2-1}\,(-\xi_\rho+\sqrt{\xi_\rho^2-1})}\,e^{s_1 t} + \frac{1}{2\sqrt{\xi_\rho^2-1}\,(\xi_\rho+\sqrt{\xi_\rho^2-1})}\,e^{s_2 t}}$$

//

case (iv): underdamped system ($\xi_\rho < 1$)

$$C(s) = \frac{\omega_n^2}{s(s^2+2\xi_\rho\omega_n s+\omega_n^2)}$$

Apply partial fraction,

$$C(s) = \frac{\omega_n^2}{s(s^2+2\xi_\rho\omega_n s+\omega_n^2)} = \frac{A}{s} + \frac{Bs+C}{s^2+2\xi_\rho\omega_n s+\omega_n^2}$$

$$\omega_n^2 = A(s^2+2\xi_\rho\omega_n s+\omega_n^2) + (Bs+C)(s)$$

$s=0$

$$\omega_n^2 = A\,\omega_n^2 \Rightarrow \boxed{A=1}$$

Expand the partial fraction eqn.

$\omega_n^2 = AS^2 + 2A\xi\omega_n S + A\omega_n^2 + BS^2 + CS$.

Equating s^2 terms.

$0 = A + B \quad \therefore \boxed{B = -1}$

Equating s terms.

$0 = 2A\xi\omega_n + C$

$0 = 2\xi\omega_n + C \quad \therefore \boxed{C = -2\xi\omega_n}$

substitute A, B, C values.

$$C(s) = \frac{1}{s} + \frac{(-s + (-2\xi\omega_n)}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Add & subtract $\xi^2\omega_n^2$ in denominator of second term.

$$C(s) = \frac{1}{s} - \left(\frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2 + \xi^2\omega_n^2 - \xi^2\omega_n^2}\right)$$

$$= \frac{1}{s} - \left(\frac{s + 2\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_n^2(1-\xi^2)}\right)$$

but $\omega_d = \omega_n\sqrt{1-\xi^2}$

$\therefore \boxed{\omega_d^2 = \omega_n^2(1-\xi^2)}$

$$\therefore C(s) = \frac{1}{s} - \left(\frac{s + 2\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2}\right)$$

$$C(s) = \frac{1}{s} - \left(\frac{s + \xi\omega_n + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} \right)$$

$$= \frac{1}{s} - \frac{s + \xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2} - \frac{\xi\omega_n(\omega_d)}{(\omega_d)(s+\xi\omega_n)^2 + \omega_d^2}$$

Taking Inverse Laplace Transform.

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_d} e^{-\xi\omega_n t} \sin\omega_d t.$$

$$c(t) = 1 - e^{-\xi\omega_n t}\left[\cos\omega_d t + \frac{\xi\omega_n}{\omega_d}\sin\omega_d t\right].$$

substitute $\boxed{\omega_d = \omega_n\sqrt{1-\xi^2}}$

$$c(t) = 1 - e^{-\xi\omega_n t}\left[\cos(\omega_n\sqrt{1-\xi^2})t + \frac{\xi\cancel{\omega_n}}{\cancel{\omega_n}\sqrt{1-\xi^2}}\sin\omega_n\sqrt{1-\xi^2}t\right]$$

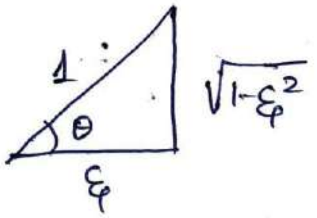
$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[(\sqrt{1-\xi^2})\cos\omega_n\sqrt{1-\xi^2}t + \xi\sin(\omega_n\sqrt{1-\xi^2})t\right.$$

Rearrange the terms,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sqrt{1-\xi^2}\cos\omega_d t + \xi\sin\omega_d t\right)$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\xi\sin\omega_d t + \sqrt{1-\xi^2}\cos\omega_d t\right).$$

On constructing right angle triangle with ξ and $\sqrt{1-\xi^2}$ we get,



$$\therefore \sin\theta = \sqrt{1-\xi^2}$$

$$\cos\theta = \xi$$

$$\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi} \Rightarrow \theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

let us express $c(t)$ in standard form,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[(\sin\omega_d t \times \cos\theta) + (\cos\omega_d t \times \sin\theta)\right]$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

where $\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$ $\quad \because \boxed{\sin(A+B) = \sin A\cos B + \cos A \sin B}$

$$\therefore \boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)}$$

— x — x —

Steady state error of second order s/m with unit step

$$e_{ss} = \lim_{s\to 0} s\left[R(s) - C(s)\right]$$

$$= \lim_{s\to 0} s\left[\frac{1}{s} - \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}\right]$$

$$= \lim_{s\to 0}\left[1 - \frac{\omega_n^2}{\omega_n^2}\right] = 0 //$$

— x — x —

SUMMARY

SECOND ORDER SYSTEM WITH IMPULSE INPUT

Sno	Damping Type	c(t)
1.	undamped ($\xi=0$)	$c(t) = \omega_n \sin\omega_n t$
2.	critically damp ($\xi=1$)	$c(t) = \omega_n^2 t e^{-\omega_n t}$
3.	overdamped ($\xi>1$)	$c(t) = \frac{\omega_n}{\sqrt{\xi^2-1}} e^{-\xi\omega_n t} \left[\sinh \omega_n\sqrt{\xi^2-1}\right] t$
4.	underdamped ($0<\xi<1$)	$c(t) = \frac{\omega_n e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_n \sqrt{1-\xi^2}\right) t$

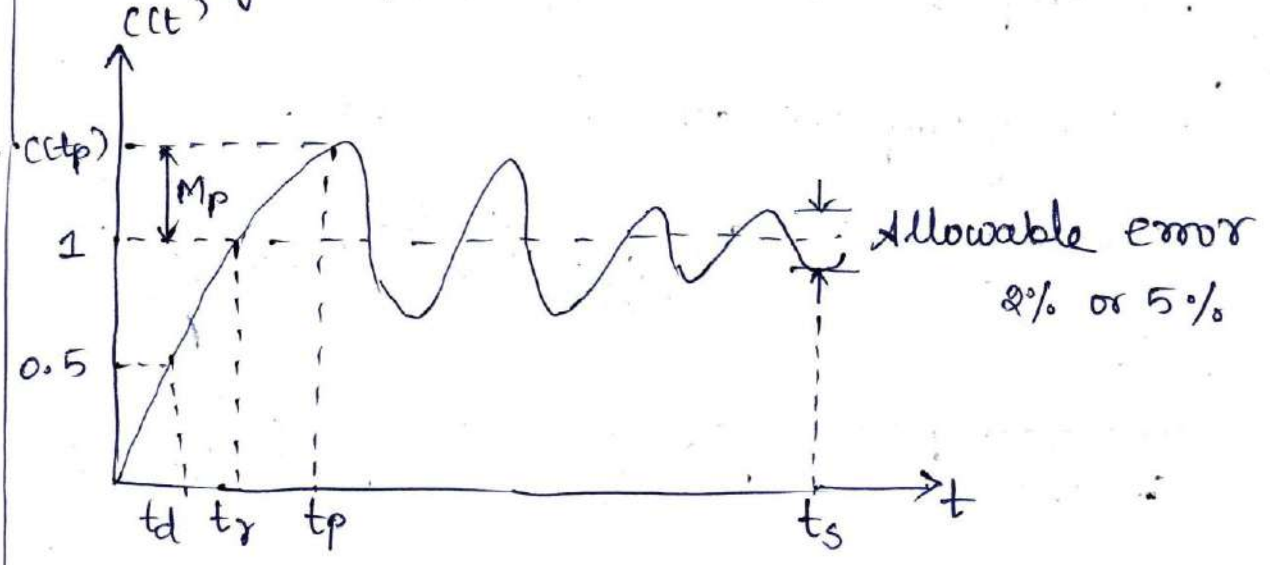
SECOND ORDER SYSTEM WITH UNIT STEP INPUT

Sno	Damping Type	c(t)
1.	undamped ($\xi=0$)	$c(t) = 1-\cos\omega_n t$
2.	critically damp ($\xi=1$)	$c(t) = 1-e^{-\omega_n t} - \omega_n t e^{-\omega_n t}$
3.	overdamped ($\xi>1$)	$c(t) = 1 + \frac{1}{2\sqrt{\xi^2-1}\left(-\xi+\sqrt{\xi^2-1}\right)} e^{s_1 t} + \frac{1}{2\sqrt{\xi^2-1}\left(\xi+\sqrt{\xi^2-1}\right)} e^{s_2 t}$
4.	underdamped ($0<\xi<1$)	$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t+\theta)$

In Both the inputs, steady state error $\boxed{e_{ss}=0}$ //.

Transient Response Specifications.

The transient response of a system to a unit step input depends on initial conditions. ∴ to compare the time response of Various System, it is necessary to start with standard initial conditions.



where, $t_d \rightarrow$ delay time

$t_r \rightarrow$ Rise time.

$t_p \rightarrow$ Peak time

$M_p \rightarrow$ Maximum peak overshoot

$t_s \rightarrow$ Settling time.

The time domain specifications are defined as follows,

(i) Delay time: (t_d):

It is the time required by the response to reach half of its final value at the first attempt.

(ii) Rise time :(t_r):

It is the time required for response to rise from 10% to 90% for overdamped or critically damped s/m and 0% to 100% for underdamped of its final value. ~~the~~ 5% to 95% of its ~~its~~ final value for may be used for critically damped system.

In general, unit step response of second order for underdamped is given by,

$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

Rise time $\Rightarrow \boxed{t = t_r}$ $\boxed{C(t_r) = 1}$

$$C(t_r) = \cancel{1} - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = \cancel{1}$$

The term $\sin(\omega_d t_r + \theta) = 0 \Rightarrow \sin\phi = 0$.

when $\phi = 0, \pi, 2\pi \dots$

$$\therefore \omega_d t_r + \theta = \pi$$

$$\boxed{t_r = \frac{\pi - \theta}{\omega_d}} = \frac{\pi - \theta}{\omega_n\sqrt{1-\xi^2}}$$

where $\omega_d = \omega_n\sqrt{1-\xi^2}$

$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$ unit is degree.

but 'π' unit is radian/~~sec~~. $\quad rad = \left(deg \cdot \frac{\pi}{180^\circ}\right)$

(iii) Peak time: (t_p) :-

Peak time is obtained by differentiating $c(t)$ with respect to t and equating to zero. At maxima, the slope is zero.

$$\therefore t_p = \left. \frac{dc(t)}{dt} \right|_{t=t_p} = 0$$

(After differentiate $c(t)$)

$$\boxed{t_p = \frac{\pi}{\omega_d}}$$

where $\boxed{\omega_d = \omega_n \sqrt{1-\xi^2}}$

"The time required for the response to reach first peak of overshoot".

(iv) Maximum peak overshoot: (M_p)

It is maximum peak value of the response measured from unity.

$$\therefore M_p = c(t_p) - 1 = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

$c(t_p) \rightarrow$ peak value of $c(t)$

$c(\infty) \rightarrow$ final value of $c(t)$.

$$\therefore \boxed{M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}}$$

(v) Settling time: (t_s):

It is the time required for response curve to reach and stay within a specified tolerance band (either 2% or 5%) of final value.

Settling time t_s ~~for 2% tolerance band~~

$$t_s = \frac{4}{\xi\omega_n} \text{ for 2\% tolerance band \& } \boxed{\xi_p = 0.76}$$

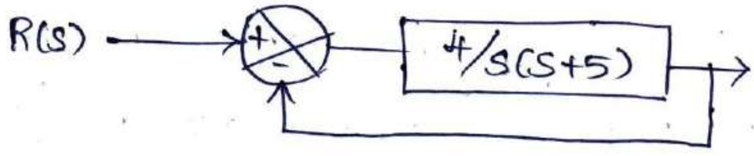
$$t_s = \frac{3}{\xi\omega_n} \text{ for 5\% tolerance band \& } \boxed{\xi_p = 0.68}$$

—— × —— × ——

Problems on Response of the system.

1. Obtain the response of unity feedback system whose open loop transfer function is $G(s) = \frac{4}{s(s+5)}$ and when input is unit step.

= Solution :-



(Always prefer negative feedback)

For unity feedback,

$$T.F = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{4/s(s+5)}{1 + 4/s(s+5)}$$

$$\frac{C(s)}{R(s)} = \frac{4}{s^2+5s+4} = \cancel{\frac{4}{s+4}}$$

$$C(s) = R(s)\left(\frac{4}{s^2+5s+4}\right) = \frac{1}{s}\left(\frac{4}{s^2+5s+4}\right)$$

$$C(s) = \frac{4}{S(S+4)(S+1)} = \frac{A}{S} + \frac{B}{S+4} + \frac{C}{S+1}.$$

$$4 = A(S+4)(S+1) + B(s)(S+1) + C(S+4)s.$$

Put $s=0 \Rightarrow \boxed{A=1}$

$s=-4 \Rightarrow \boxed{B=1/3}$

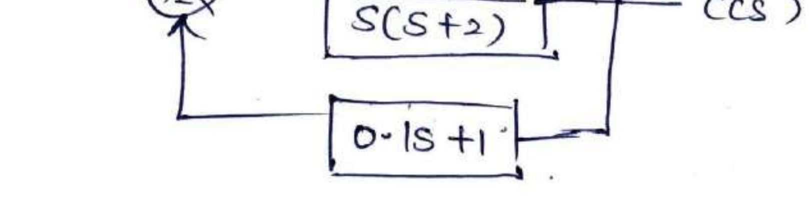
$s=-1 \Rightarrow \boxed{C=-4/3}$

$$\therefore\ C(s) = \frac{1}{S} + \frac{1}{3(S+4)} - \frac{4}{3(S+1)}$$

Take inverse Laplace transform,

$$\boxed{C(t) = 1 + \frac{1}{3}e^{-4t} - \frac{4}{3}e^{-t}}.$$ //

2. A position control system with velocity feedback is shown in figure. Calculate rise time, peak time, peak overshoot, settling time and also sketch the response.



= solution:-

$$T.F = \frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$$

$$\frac{C(S)}{R(S)} = \frac{100/S(S+2)}{1+\left(100/S(S+2)\right)(0.1S+1)}$$

$$\boxed{\frac{C(S)}{R(S)} = \frac{100}{S^2+12S+100}} \rightarrow (1)$$

General formula for 2nd order system,

$$\boxed{\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}} \rightarrow (2)$$

compare (1) & (2), $\therefore \omega_n^2 = 100 \Rightarrow \boxed{\omega_n = 10 \text{rad/sec}}$

$2\xi\omega_n = 12 \Rightarrow \boxed{\xi = 0.6}$ no unit

(i) rise time :-

$$t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}$$

$$\theta = \tan^{-1}\left(\sqrt{1-\xi^2}/\xi\right)$$

$$\theta = \tan^{-1}\left(\sqrt{1-0.6^2}/0.6\right) = \tan^{-1}(1.333) = 53.13^\circ$$

$$\theta = 53.13 \times \frac{\pi}{180} = 0.92 \text{ radian.}$$

$\omega_d = \omega_n\sqrt{1-\xi^2} = 8$ rad/sec $\quad \therefore t_r = \frac{\pi - 0.92}{8} = 0.27$ sec

(ii) peak time: $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{8} = 0.39$ sec.

(iii) Max. peak overshoot: $M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$

$$= e^{-0.6\pi/\sqrt{1-0.6^2}}$$

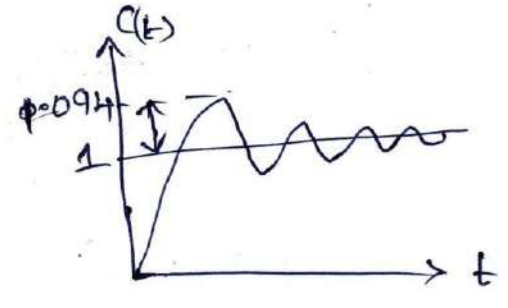
$$\boxed{\begin{array}{l} M_p = 0.094 \\ \%M_p = 9.4\% \end{array}}$$

(iv) settling time (t_s) :-

$2\%\ t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.6\times10} = 0.66$ sec.

$5\%\ t_s = \frac{3}{\xi\omega_n} = \frac{3}{0.6\times10} = 0.5$ sec.

(v) Response :-



3) The response of a servomechanism is $c(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$ when subject to a unit step input. obtain an expression for closed loop transfer function. Determine the undamped natural frequency and damping ratio.

= Solution:-

$$c(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$$

Take laplace transform,

$$C(s) = \frac{1}{s} + \frac{0.2}{s+60} - \frac{1.2}{s+10}$$

$$C(s) = \frac{(s+60)(s+10) + 0.2s(s+10) - (1.2)s(s+60)}{s(s+60)(s+10)}$$

$$C(s) = \frac{600}{s(s+60)(s+10)}$$

Input : unit step signal, $R(s) = 1/s$.

$$C(s) = \frac{1}{s}\left(\frac{600}{s(s+60)(s+10)}\right)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{600}{s^2+70s+600}}$$

In general, $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

compare, $\omega_n^2 = 600 \Rightarrow \boxed{\omega_n = 24.49 \text{ rad/sec}}$

$$2\xi\omega_n = 70$$

$$\boxed{\xi = 1.42}$$

$\therefore$ undamped natural freq $= \omega_n = 24.49$ rad/sec.

Damping ratio $= \xi = 1.42$ //

4) The unity feedback system is characterized by an open loop transfer function $G(s) = K/s(s+10)$. Determine gain K, so that system will have damping ratio of 0.5 for this value of K. Determine settling time, peak overshoot and time at peak overshoot for a unit step input.

= Solution: $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$

$$= \frac{K/s(s+10)}{1+K/s(s+10)} = \frac{K}{s^2+10s+K}$$

by comparing with $\Rightarrow \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

we get, $\omega_n^2 = K$; $2\xi\omega_n = 10$ $\quad \xi = \frac{10}{2\omega_n} = \frac{10}{2\sqrt{K}}$

$\boxed{\omega_n = \sqrt{K}}$ $\quad \xi = 0.5 \rightarrow$ given.

$\xi = \frac{10}{2\sqrt{K}} \Rightarrow 0.5 = \frac{5}{\sqrt{K}}$

$\boxed{K = 100}$ $\quad \therefore \boxed{\omega_n = 10 \text{ rad/sec}}$

Peak overshoot :- $M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.163$

$\% M_p = 16.3\%$

Peak time :- $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 0.363 \text{ sec}$ //

5) The open loop transfer function of a unity feedback system is given by $G(s) = K/s(sT+1)$, where K & T are positive constant. By what factor should the amplifier gain K be reduced, so that peak overshoot of unit step response of the system is reduced from 75% to 25%.

⇒ Solution: $$\frac{C(s)}{R(s)} = \frac{K/s(sT+1)}{1+K/s(sT+1)} = \frac{K}{Ts^2+s+K} = \frac{K/T}{s^2+1/T\,s+K/T}$$

compare, $$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi_p\omega_n s+\omega_n^2} = \frac{K/T}{s^2+1/T\,s+K/T}$$

on comparing, we get,

$$\omega_n^2 = K/T \quad ; \quad 2\xi_p\omega_n = 1/T$$

$$\omega_n = \sqrt{K/T} \qquad \xi_p = \frac{1}{2\sqrt{\frac{K}{T}}\,T} = \frac{1}{2\sqrt{KT}}$$

$$\boxed{\omega_n = \sqrt{K/T}\ ;\ \xi_p = 1/2\sqrt{KT}}$$

peak overshoot M_p is reduced (75% to 25%) by increasing ξ_p. The ξ_p is increased by reducing the K.

$$\boxed{M_p = e^{-\xi_p\pi/\sqrt{1-\xi_p^2}}}$$

Taking natural logarithm on both sides,

$$\ln M_p = -\xi_p\pi/\sqrt{1-\xi_p^2}$$

Squaring on both sides,

$$(\ln M_p)^2 = \xi_p^2\pi^2 / 1-\xi_p^2$$

$$(\ln M_p)^2 (1-\xi_p^2) = \xi_p^2\pi^2.$$

$$(\ln M_p)^2 - \xi_p^2 (\ln M_p)^2 = \xi_p^2\pi^2$$

$$(\ln M_p)^2 = \xi_p^2 (\pi^2 + (\ln M_p)^2)$$

$$\xi_p^2 = \frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}$$

but $\xi_p = \frac{1}{2\sqrt{KT}} \Rightarrow \boxed{\xi_p^2 = \frac{1}{4KT}}$

$$\therefore \frac{1}{4KT} = \frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}$$

$$\boxed{K = \frac{\pi^2 + (\ln M_p)^2}{4T(\ln M_p)^2}}$$

At $M_p = 0.75$, $K = K_1 = \frac{\pi^2 + (\ln 0.75)^2}{4\pi (\ln 0.75)^2}$

$$\boxed{K_1 = 30.1/T}$$

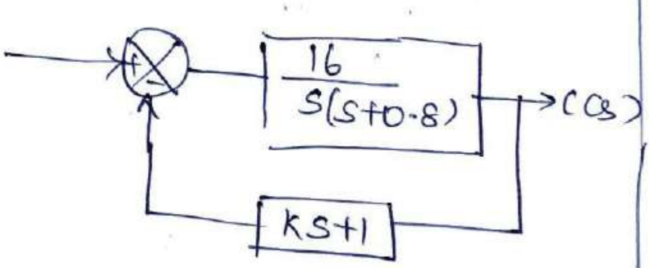
At $M_p = 0.25$, $K = K_2 = \frac{\pi^2 + (\ln 0.25)^2}{4\pi (\ln 0.25)^2}$

$$\boxed{K_2 = 1.53/T}$$

$\therefore \frac{K_1}{K_2} = 19.6 \Rightarrow K_2 = \frac{K_1}{19.6} \Rightarrow$ Gain k is reduced by 19.6 times the first gain//

6) A position control system with velocity feedback as shown. Given that $\xi_p = 0.5$. Also calculate rise time, peak time, maximum overshoot and settling time.

= Solution :-



$$T.F = \frac{C(S)}{R(S)} = \frac{16/S(S+0.8)}{1+\left(16/S(S+0.8)\right)(KS+1)}$$

$$T.F = \frac{16}{S^2+(0.8+16K)S+16}$$

on comparing we get,

$\omega_n^2 = 16$

$\boxed{\omega_n = 4\,rad/sec}$

$0.8+16K = 2\xi_p\omega_n$.

$\xi_p = 0.5$ (given)

$\therefore \boxed{K = 0.2}$

$$T.F = \frac{C(S)}{R(S)} = \frac{16}{S^2+4S+16}$$

(i) Damped frequency : $\omega_d = \omega_n\sqrt{1-\xi_p^2} = 3.464\ rad/sec$.

(ii) rise time : $t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-1.047}{3.464} = 0.6046.$

(Assume $\theta = 1.047$) $\Rightarrow \theta = \tan^{-1}\sqrt{1-\xi_p^2}/\xi_p$

(iii) peak time : $t_p = \frac{\pi}{\omega_d} = 0.907$ sec.

(iv) $M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.163$

$$\boxed{\% M_p = 16.3\%}$$

(v) Settling time :-

For 2% error, $t_s = \frac{4}{\xi\omega_n} = 2sec$

For 5% error, $t_s = \frac{3}{\xi\omega_n} = 1.5sec$.

7) A unity feedback control system is characterized by following open loop transfer function $G(s) = (0.4s+1)/s(s+0.6)$. Determine transient response for unit step input.

= Solution :-

$$T.F = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$ 1

$$\frac{C(s)}{R(s)} = \frac{(0.4s+1)/s(s+0.6)}{1+((0.4s+1)/s(s+0.6))} = \frac{0.4s+1}{s^2+s+1}$$

$$C(s) = \left(\frac{1}{s}\right)\left(\frac{0.4s+1}{s^2+s+1}\right)$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+s+1}$$

$$0.4s+1 = A(s^2+s+1) + (Bs+C)s.$$

$\boxed{A=1}$ when $s=0$,

Equating s^2 terms, $0 = A+B \Rightarrow \boxed{B=-1}$

Equating s term, $0.4 = A + C$ $\therefore \boxed{C = -0.6}$

$$C(s) = \frac{1}{s} - \frac{s+0.6}{s^2+s+1} = \frac{1}{s} - \frac{s+0.6}{(s^2+2\times0.5s+0.5^2)+0.75}$$

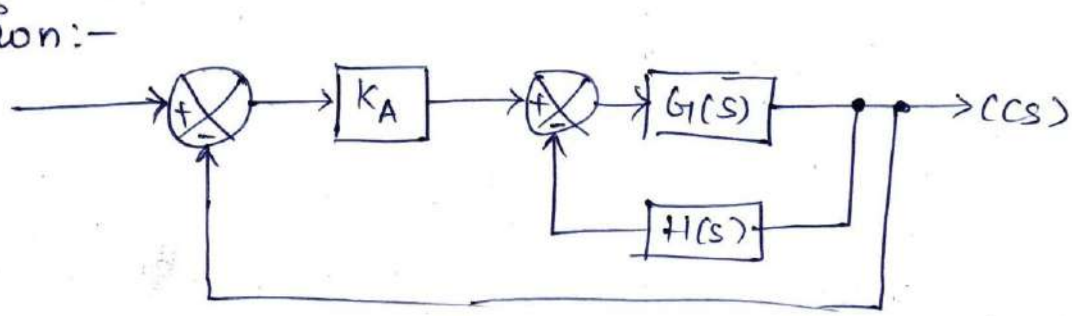
$$C(s) = \frac{1}{s} - \frac{s+0.5}{(s+0.5)^2+0.75} - \frac{0.1}{\sqrt{0.75}}\frac{\sqrt{0.75}}{(s+0.5)^2+0.75}$$

Take inverse L.T,

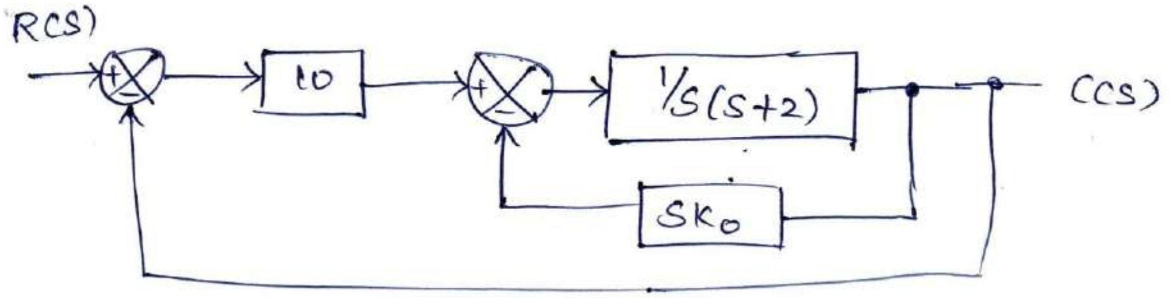
$$c(t) = 1 - e^{-0.5t}\cos\sqrt{0.75}\,t - \frac{0.1}{\sqrt{0.75}}e^{-0.5t}\sin\sqrt{0.75}\,t$$

8. A unity feedback control system has an amplifier with gain $K_A = 10$ and gain ratio, $G(s) = 1/s(s+2)$ in feedforward path. A derivative feedback $H(s) = sK_0$ is introduced as a minor loop around $G(s)$. Determine derivative feedback constant K_0, so that the system damping factor is 0.6.

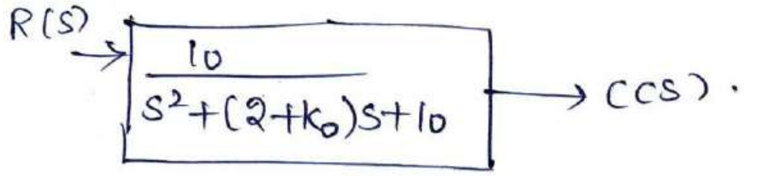
= solution:-



where $K_A = 10$, $G(s) = \frac{1}{s(s+2)}$, $H(s) = sK_0$.



⇓ After simplification



$$\therefore \quad \frac{C(S)}{R(S)} = \frac{10}{S^2+(2+K_0)S+10}$$

On compare, $\dfrac{C(S)}{R(S)} = \dfrac{\omega_n^2}{S^2+2\xi_p\omega_n S+\omega_n^2}$.

$\omega_n^2 = 10 \Rightarrow \omega_n = \sqrt{10} = 3.162$ rad/sec.

$2 + K_0 = 2\xi_p\omega_n$.

$K_0 = 2 \times 0.6 \times 3.162 - 2 = 1.7944$.

$$\boxed{K_0 = 1.7944}$$

9) A closed loop servo is represented by the differential equation $\frac{d^2c}{dt^2} + 8\frac{dc}{dt} = 64e$. where c is displacement of output shaft, r is displacement of input shaft and $e = r - c$. Determine undamped natural frequency, damping ratio, M_p for unit step input of 12 units.

= Solution :-

The mathematical equation governing the system are,

$$\frac{d^2c}{dt^2} + 8\frac{dc}{dt} = 64e.$$

put $\boxed{e = r - c}$ $\Rightarrow \frac{d^2c}{dt^2} + 8\frac{dc}{dt} = 64(r - c)$.

Taking Laplace transform,

$$s^2 C(s) + 8sC(s) = 64[R(s) - C(s)]$$

$$[s^2 + 8s + 64]C(s) = 64R(s)$$

$$\therefore \quad T.F = \frac{C(s)}{R(s)} = \frac{64}{s^2+8s+64}.$$

Now, compare with second order s/m.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$\therefore$ Natural undamped frequency : $\boxed{\omega_n = 8 \text{ rad/sec}}$

Damping ratio : $\boxed{\xi = 0.5}$

$$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} = 0.163.$$

and it input is $\boxed{12 \text{ units}}$ $\therefore M_p = 0.163 \times 12 = 1.956$

$$\% M_p = 1.956 \times 100 = 195.6\%$$

— x — x —